Interdisciplinary Division of Aeronautical and Aviation Engineering 航空工程跨領域學部

The Hong Kong Polytechnic University 香港理工大學

# Satellite Navigation

## AAE4203 – Guidance and Navigation

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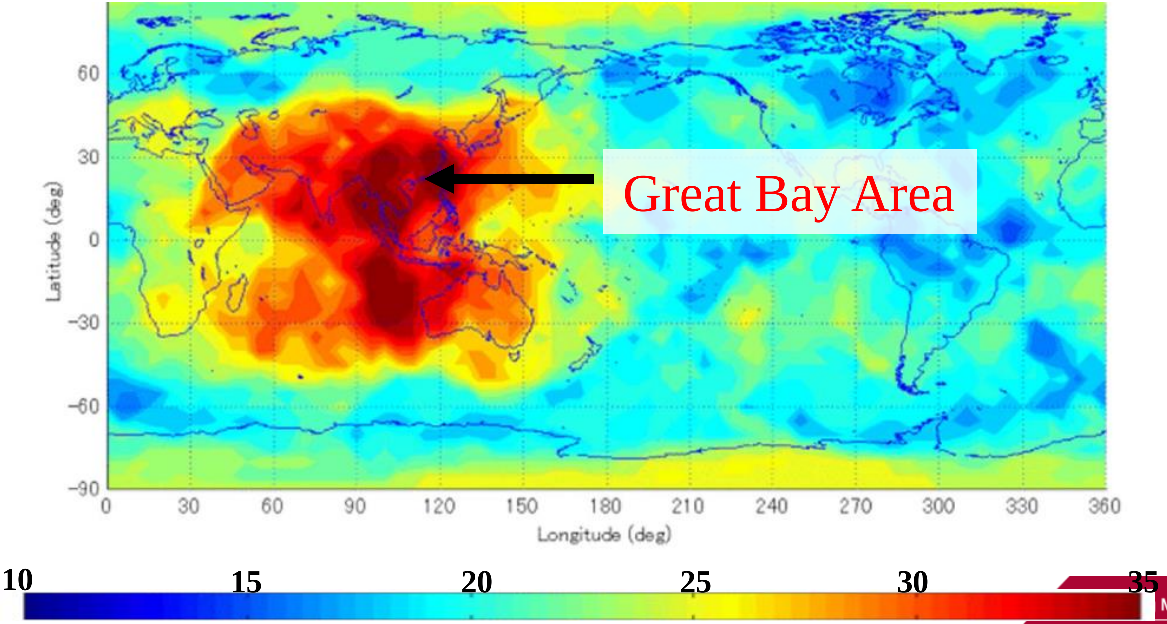
*Week 2~3, 19, 26 Jan 2022*

# Global Navigation Satellite System (GNSS)

> US Global Positioning System GPS
> Russia GLONASS
> China Beidou
> EU Galileo

# New Era of GNSS

> GPS, GLONASS, Galileo and Beidou
> Visible GNSS satellites with mask angle > 30º in 2020



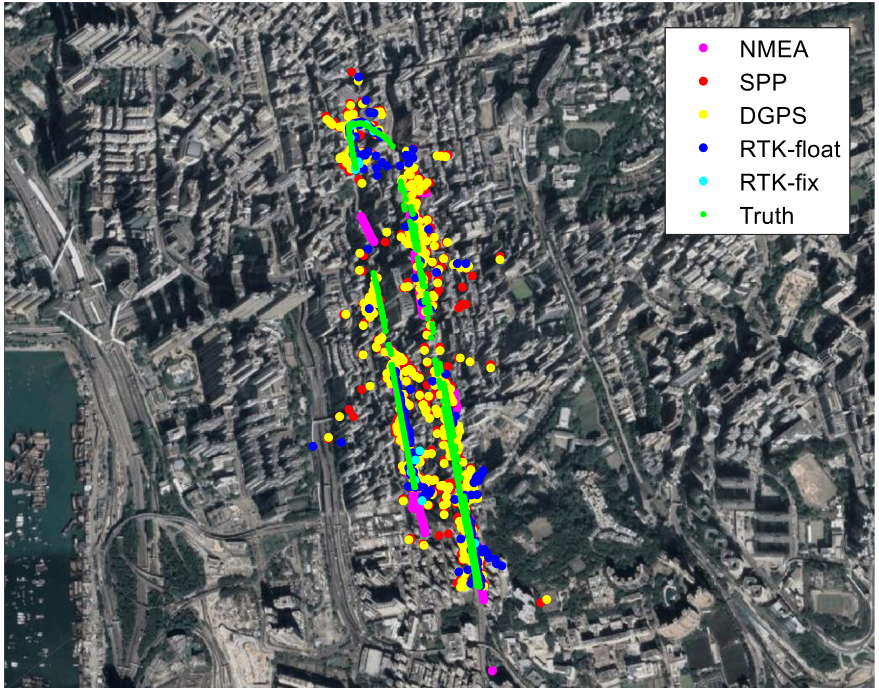
# GNSS in Autonomous Driving

| Model |  |  |  |  |  |
|---|---|---|---|---|---|
| Company Name | Waymo | Ford | Tesla | Nuro | Oxbotica |
| Positioning Solution | GNSS-RTK/INS/LiDAR/HD Map | GNSS-RTK/INS/LiDAR/HD Map | GNSS/INS/LiDAR/HD Map | GNSS-RTK/INS/LiDAR/HD Map | GNSS-RTK/INS/LiDAR/HD Map |
| Tested Scenario | Open area/sub-urban | Open area/sub-urban | Open area/sub-urban | Open area/sub-urban | Open area/sub-urban |
| Level of Automation | L5 | L5 | L2.5 | L5 | L5 |

ADAS*: Advanced Driving Assistant System

# GNSS in Urban Canyons

SPP*: Single Point Positioning
RTK*: Real-time Kinematic




| nSat | Full nSat | Ratio | HDOP | VDOP | XDOP | YDOP |
|---|---|---|---|---|---|---|
| 8.50 | 21.70 | 39.03% | 2.82 | 4.69 | 1.87 | 1.85 |

| Type | Availability | 2D Error | X Error | Y Error | 2D STD | X STD | Y STD |
|---|---|---|---|---|---|---|---|
| NMEA | 91.43% | 84.74 | 78.75 | 16.71 | 85.11 | 87.68 | 16.01 |
| SPP | 76.94% | 51.49 | 31.71 | 32.28 | 61.28 | 49.48 | 43.73 |
| DGNSS | 72.07% | 45.87 | 28.91 | 28.62 | 57.41 | 46.54 | 39.74 |
| RTK FLOAT | 31.40% | 28.91 | 14.69 | 21.66 | 44.59 | 30.13 | 35.10 |
| RTK FIX | 2.09% | 10.10 | 5.88 | 7.12 | 12.87 | 9.89 | 9.25 |

# Background

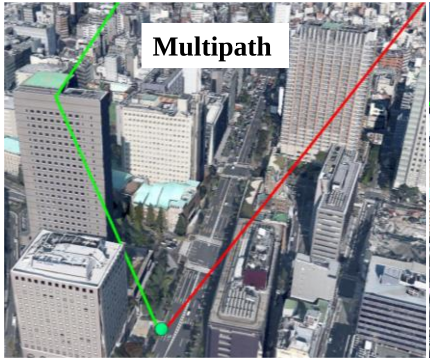
NLOS*: Non-line-of-sight

Problem 1: Poor GNSS measurements quality:

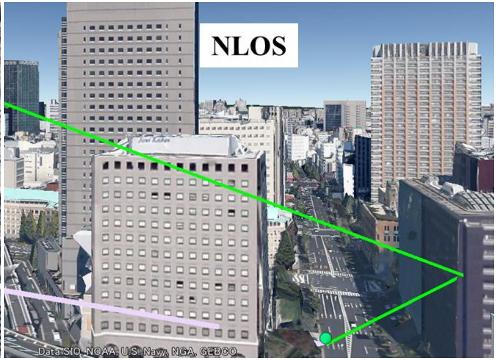
- NLOS receptions
- Multipath effects
- …

Problem 2: Poor satellite geometry:

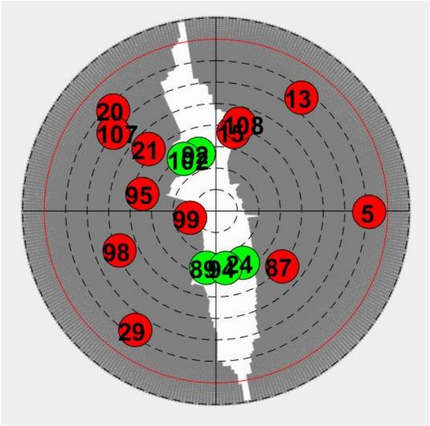
- Limited satellites numbers
- Hard to get fixed during ambiguity resolution




Hsu, 2016




Hsu, 2016




Satellite geometry in Urban

[1]J. Breßler, etc, "GNSS positioning in non-line-of-sight context—A survey," *ITSC 2016*.
[2] Hsu, Li-Ta, etc. "3D building model-based pedestrian positioning method using GPS/GLONASS/QZSS and its reliability calculation." *GPS solutions*, 2016

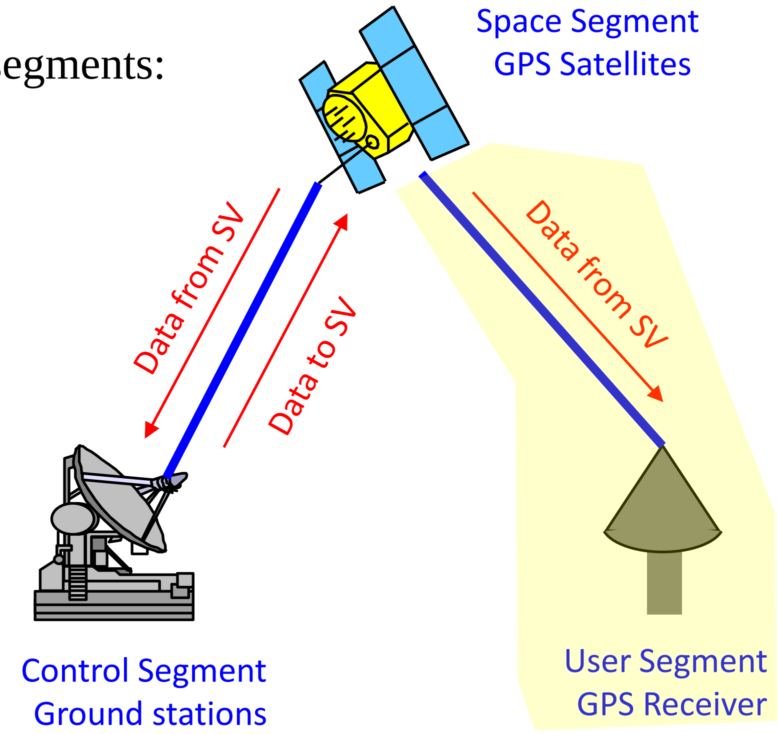
# Outline

- GPS Overview
- Receiver Position Estimation
- GPS Performance
- Improved GPS Performance

# GPS Overview

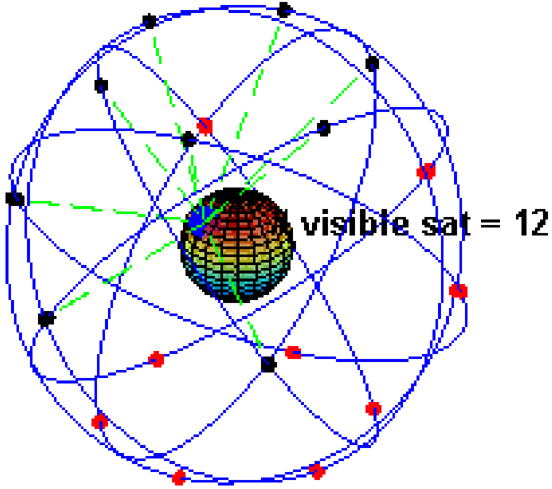
# System Configuration of GPS

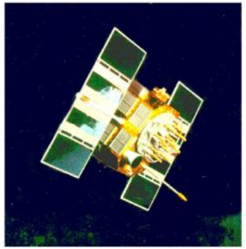
> Satellite navigation system is consisted of three segments:
1. Control segment
   - Command infrequent small maneuvers to maintain orbit
   - Keep the synchronization of GPS time
2. Space segment (broadcasting)
   - 31+ medium earth orbit (MEO) satellites
   - 6 orbit planes
3. User segment
   - Antenna
   - A/D converter
   - Signal processing
   - **<u>Positioning algorithm</u>**



# Space Segment

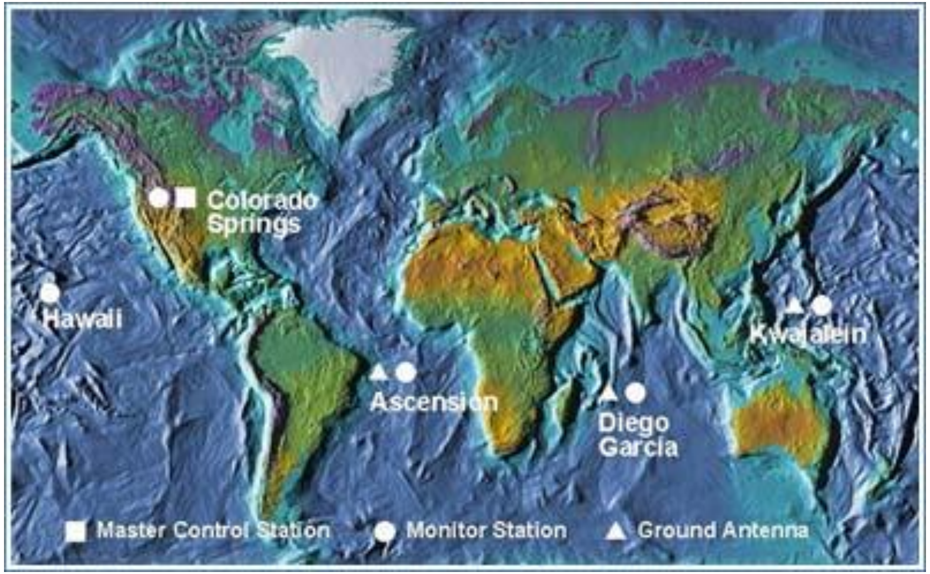
- Able to see 5 to 8 satellites at any point on the earth
- Each satellite has atomic clocks
- 32 satellites in 6 orbital planes (5-6 satellite per orbit)
- 20,200 km altitude, 55 degree inclination
- Two revolutions per sidereal day
- One sidereal day is 23 hours 56 minutes 4.091seconds
- SVs repeat more or less the same ground track on each day



# Control Segment

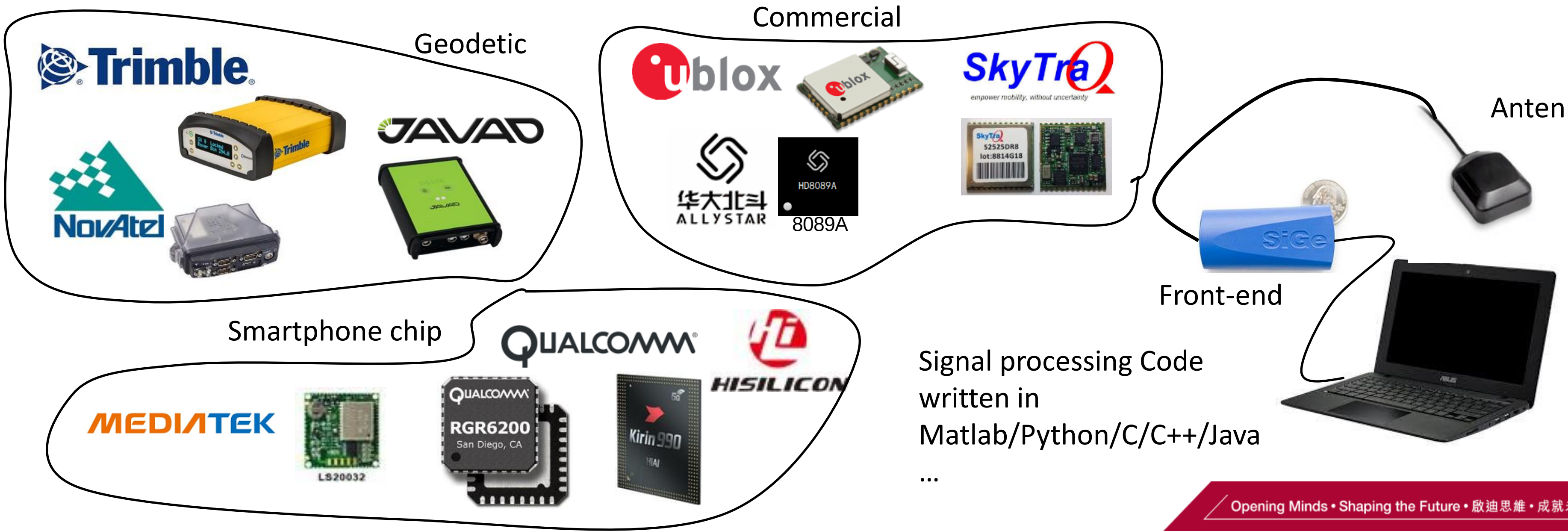
- Monitor stations measure signals from SVs and compute precise orbital and clock corrections data for each SV.
- Master Control station uploads orbital & clock data to SVs.



# User Segment

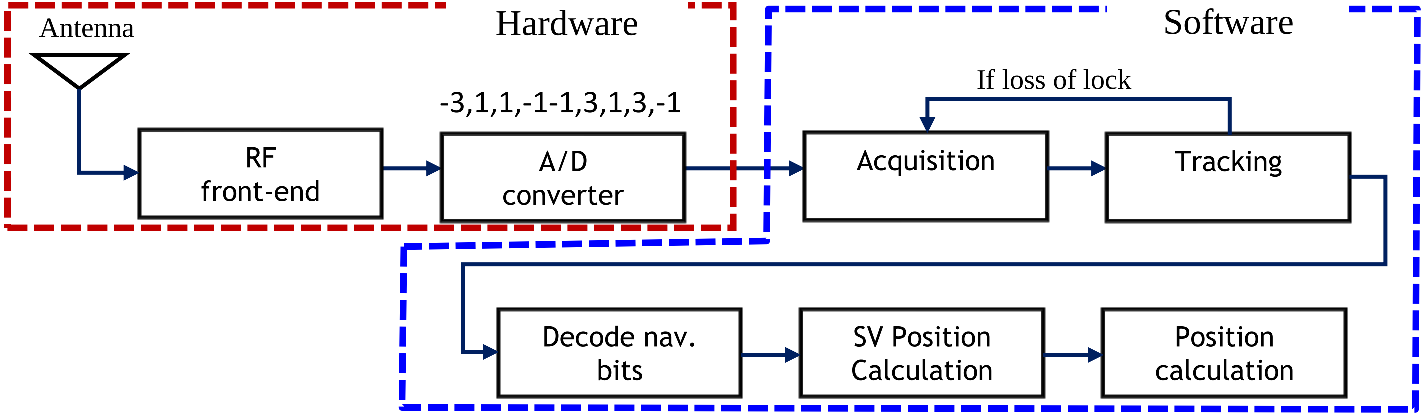
> GPS receivers with quartz clocks can convert SV signals into position and time estimates and derive velocity.

> Four satellites are required to compute the four dimensions of X, Y, Z (position) and Time.




Signal processing Code written in Matlab/Python/C/C++/Java …

# Architecture of GPS Software Define Receiver




> **Acquisition:** to determine visible satellites, coarse values of carrier frequency, and code delay of received signals.

> **Tracking:** to refine these values and keep track and demodulate navigation data from satellites.

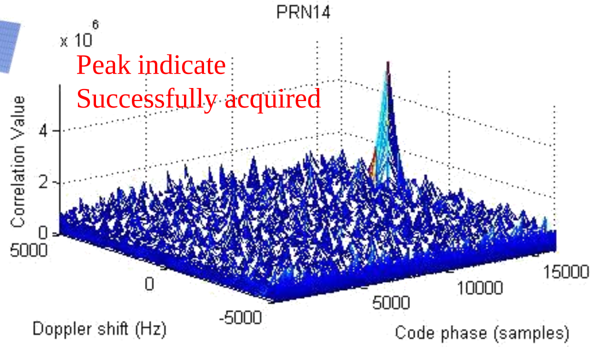
> **Navigation Data Decode**: to obtain Pseudorange, GPS time, Ephemeris, Almanac, and Klobuchar information.

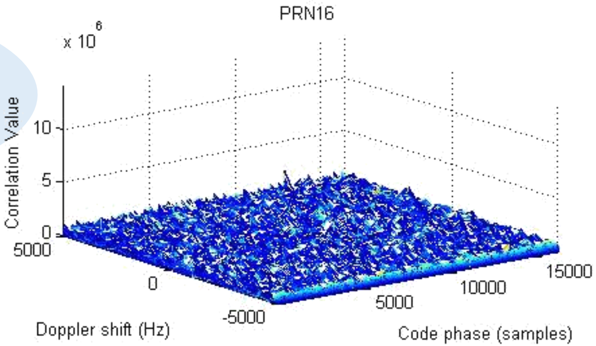
> **User Positioning:** to calculate the receiver position via estimating technique.

# What is signal acquisition?

Lots of satellite in the sky! Which is this one!?

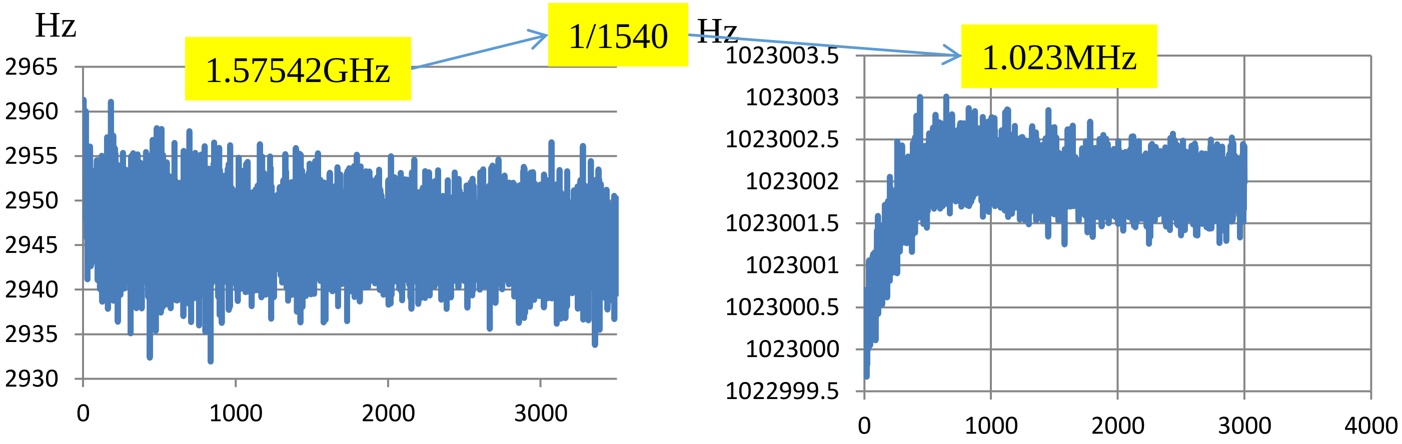
Don't worry! Acquisition process could help you to identify which satellite it is!



# Purpose of Tracking

> Tracking is to continuously track the code-phase and Doppler frequency of GNSS signals. Loop filter is used in the tracking loop.




Doppler Frequency (ms) Code Frequency (ms)

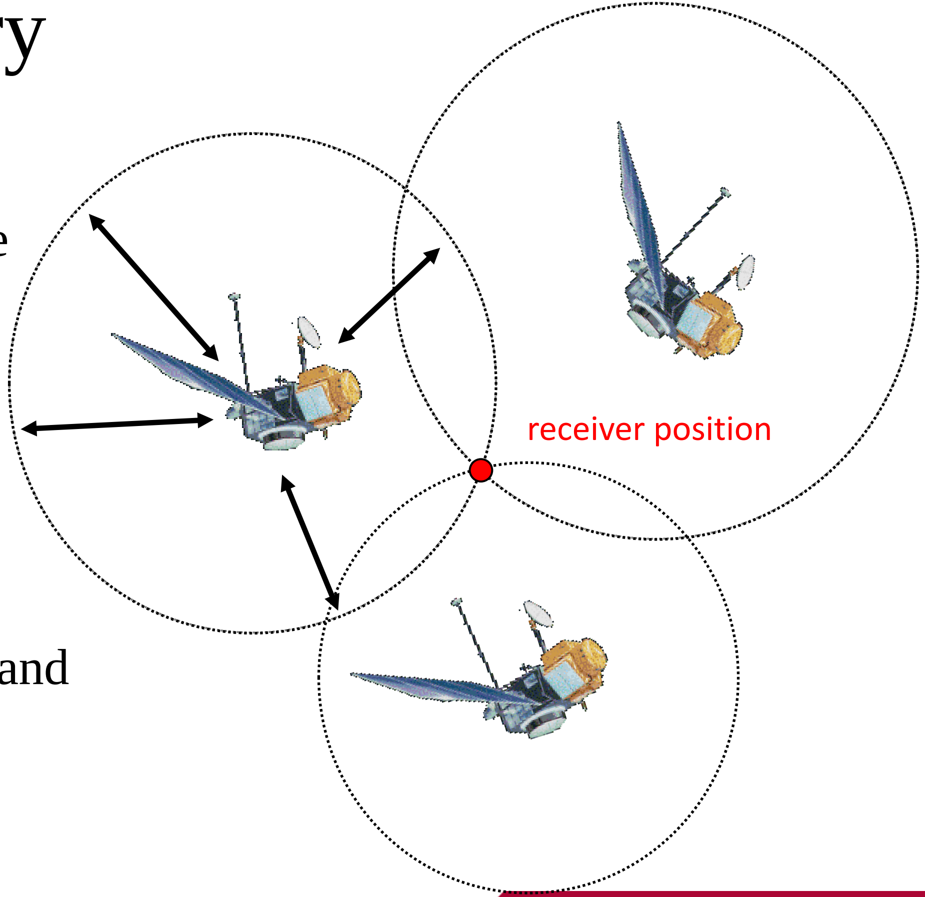
# GNSS Positioning Theory

> GNSS Positioning is based on the triangulation method.

> Known information obtained from the signal processing
- Position of satellites
- Distance between satellites and receiver (Pseudoranges)

> The time difference between satellite and receiver is also estimated in the positioning process.

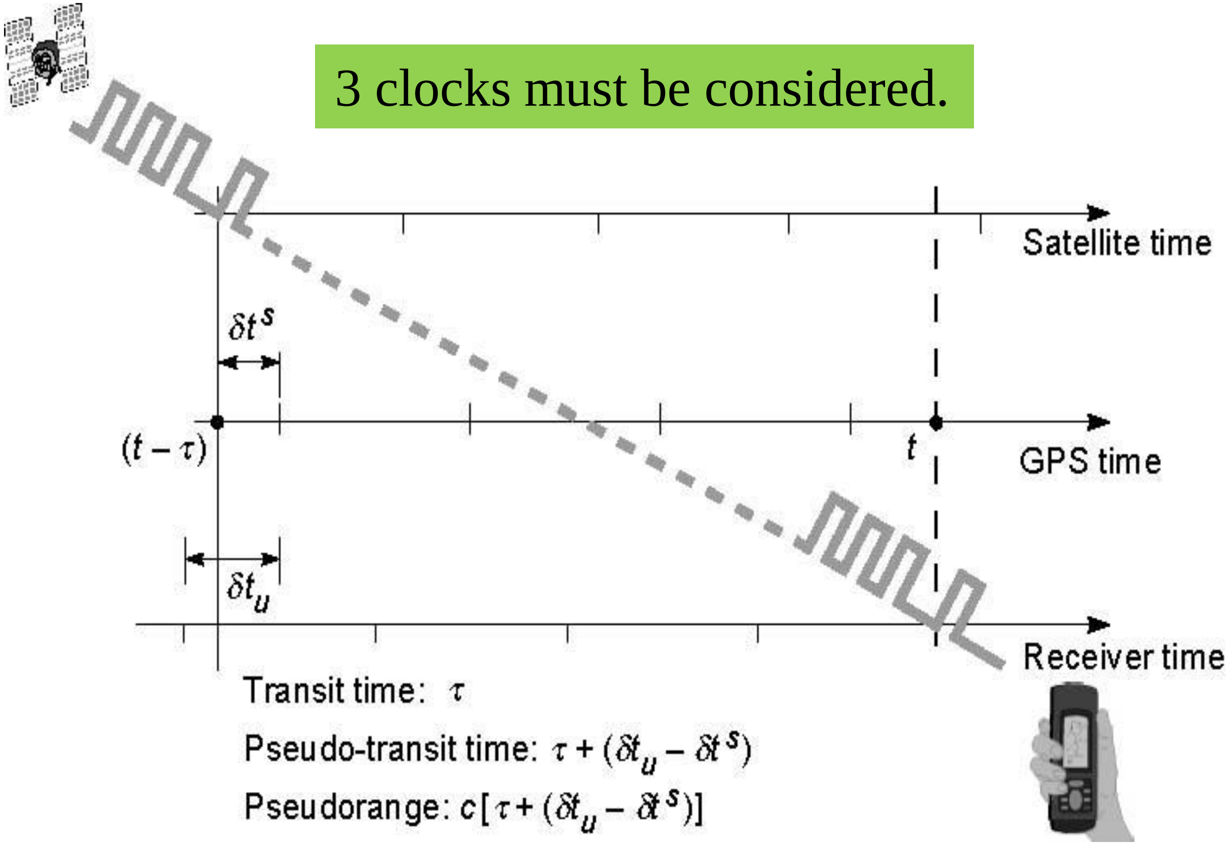
> At least 4 satellites are required.



# Receiver Position Estimation

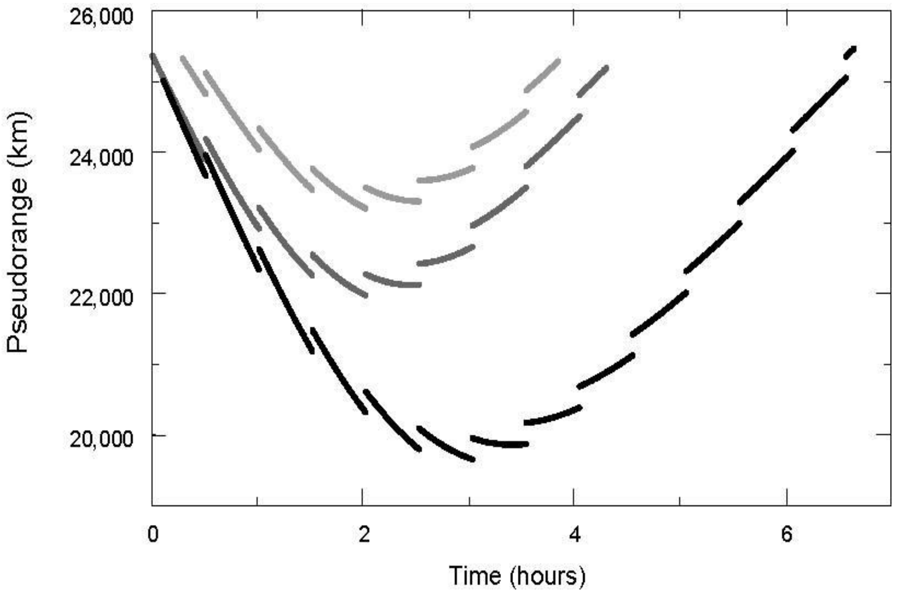
# Code Phase Measurement (Pseudorange)

3 clocks must be considered.




3 clocks are not synchronized.
Satellite clock error can be corrected using navigation message.
User clock error can be estimated as an unknown parameter in the positioning.

# Real Pseudorange Measurements




The variations of pseudo-range are mainly due to the satellite motion and earth rotation. Several gaps in all satellites are due to receiver clock offset. Receiver usually offset their own clock because the receiver clock error continues to increase.

# Position Estimation

> Satellite position in the transmitted time "t - τ".

> Pseudo-range between satellite and user in the received time "t"

$$\rho^{(k)}(t) = r^{(k)}(t, t-\tau) + c\left[\delta t_u(t) - \delta t^{(k)}(t-\tau)\right] + I^{(k)}(t) + T^{(k)}(t) + \varepsilon_\rho^{(k)}(t)$$

Clock Errors

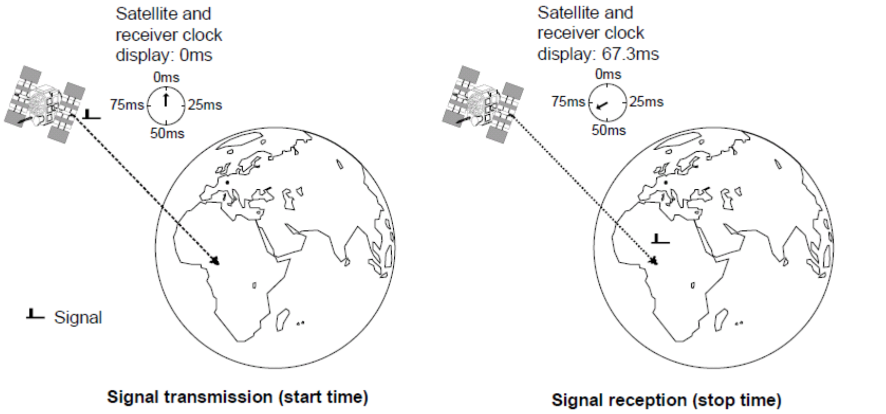
The reason why we call "pseudo-range" is from second term.

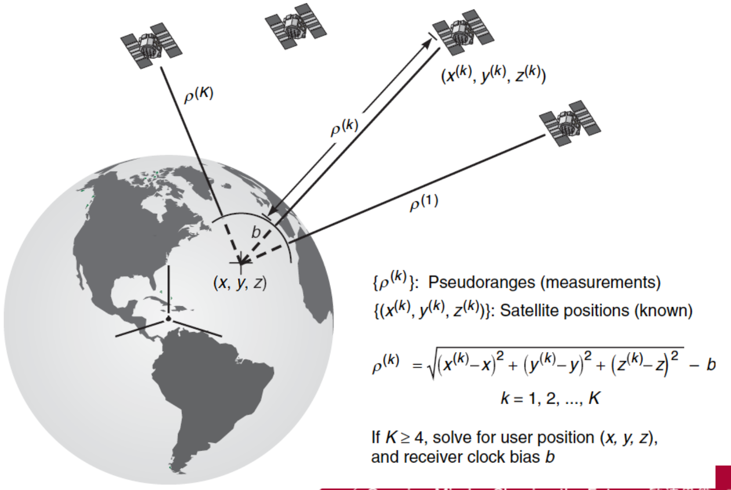
Satellite clock and Receiver clock are not synchronized.

How many unknown parameters do we have ?

# x, y, z, receiver clock offset

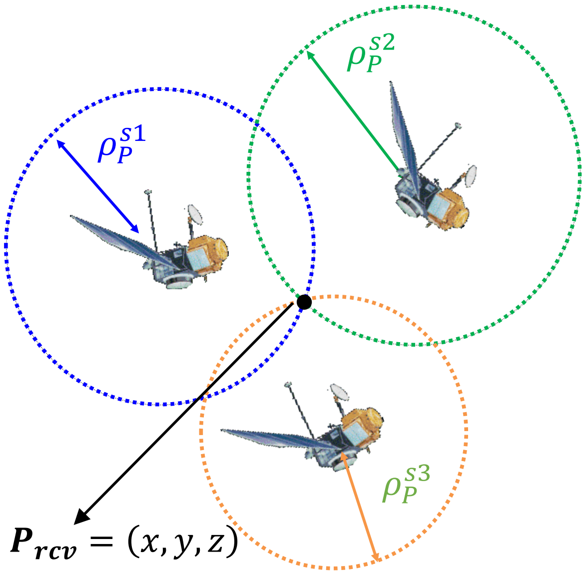
- Satellite clock is corrected using navigation data.
- Fortunately, receiver clock offset is same for all satellites.
- Therefore, unknown variables should be solved are x, y, z and receiver clock offset.



# Triangulation Positioning Theory

Goal: Solve x,y,z!




Known Information

$$\rho_P^{s1} = \sqrt{(x^{s1}-x)^2 + (y^{s1}-y)^2 + (z^{s1}-z)^2} + b$$

$$\rho_P^{s2} = \sqrt{(x^{s2}-x)^2 + (y^{s2}-y)^2 + (z^{s2}-z)^2} + b$$

$$\rho_P^{s3} = \sqrt{(x^{s3}-x)^2 + (y^{s3}-y)^2 + (z^{s3}-z)^2} + b$$

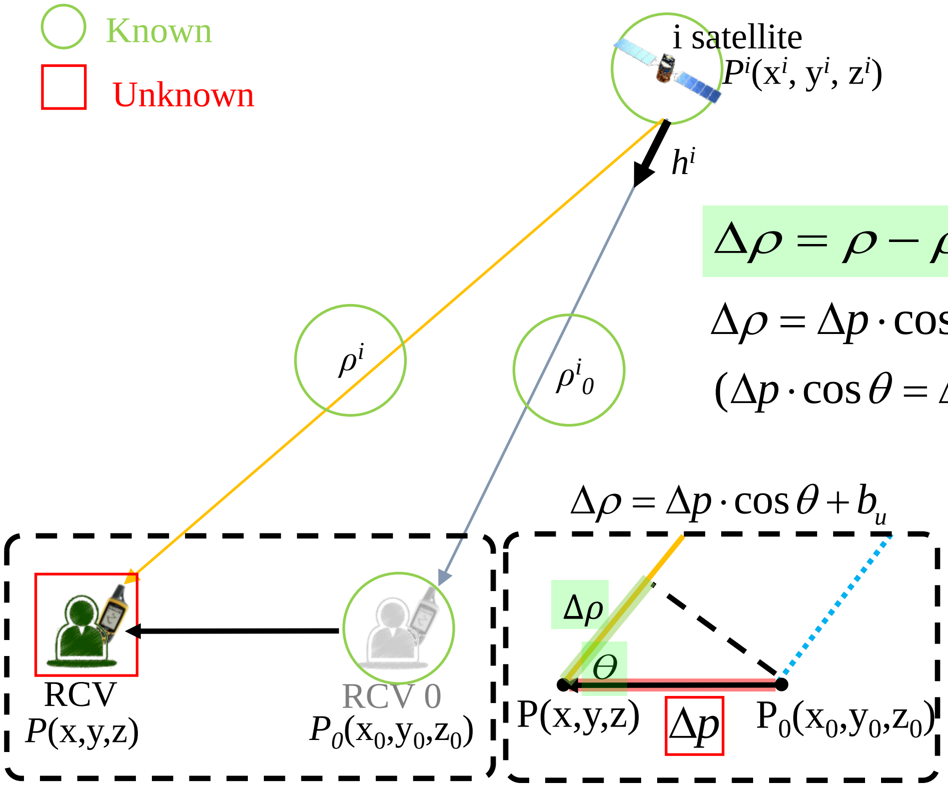
Pseudorange measurement

Satellite Positions

3 equations with 4 unknowns! Therefore, 4 satellites are required

Can we solve? YES! How!? Mathematically, linearize the equation by Taylor Series Expansion at a point we GUESS.

# Positioning using Least Square Estimation




$$\Delta\rho = \rho - \rho_0 = (\rho_{true} + b_u) - \rho_0$$

$$\Delta\rho = \Delta p \cdot \cos\theta + b_u$$

$$(\Delta p \cdot \cos\theta = \Delta\overline{p} \times 1 \times \overline{\theta}_{uni})$$

# Positioning using Least Square Estimation

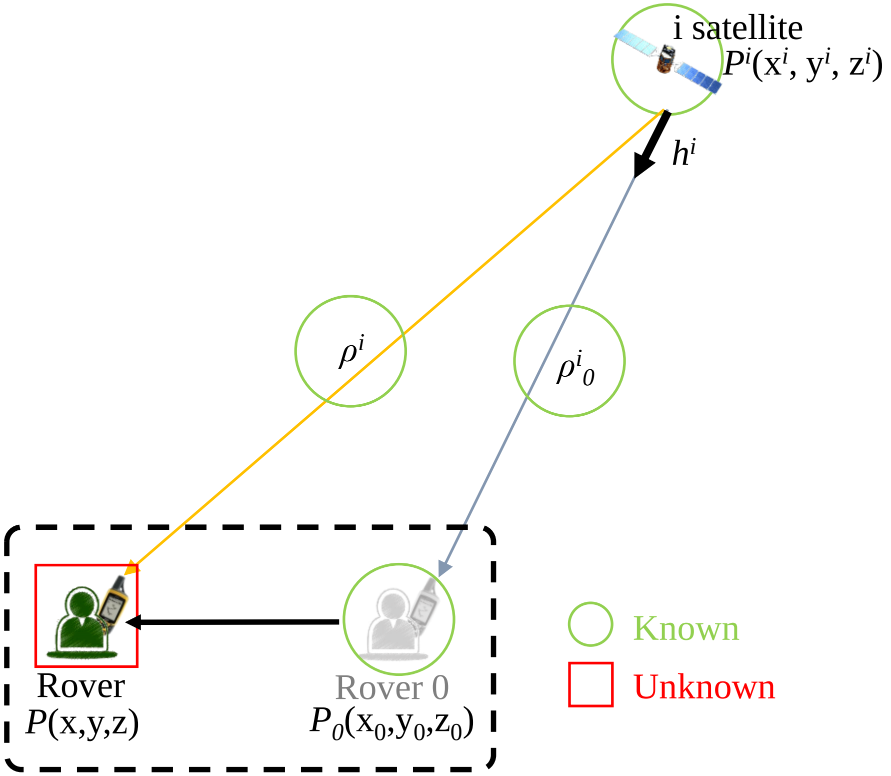
$$\Delta\rho = \rho - \rho_0 = (\rho_{true} + b_u) - \rho_0$$

$$\Delta\rho = \Delta p \cdot \cos\theta + b_u \qquad (\Delta p \cdot \cos\theta = \Delta\bar{p} \times 1 \times \bar{\theta}_{uni})$$

$$\Delta\rho^i = \begin{bmatrix} \frac{(x^i - x_0)}{\rho^i{}_0} & \frac{(y^i - y_0)}{\rho^i{}_0} & \frac{(z^i - z_0)}{\rho^i{}_0} & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta b \end{bmatrix}$$

$$\Delta\rho = G\Delta p \qquad (\Delta p = G^{-1}\Delta\rho)$$

$$\begin{bmatrix} \Delta p_x \\ \Delta p_y \\ \Delta p_z \\ \Delta b_u \end{bmatrix} = \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \\ b_u - b_{u,0} \end{bmatrix} \qquad \begin{bmatrix} x_n \\ y_n \\ z_n \\ b_{u,n} \end{bmatrix} = \begin{bmatrix} x_{n-1} \\ y_{n-1} \\ z_{n-1} \\ b_{u,n-1} \end{bmatrix} + \begin{bmatrix} \Delta p_{x,n-1} \\ \Delta p_{y,n-1} \\ \Delta p_{z,n-1} \\ \Delta b_{u,n-1} \end{bmatrix}$$




https://en.wikipedia.org/wiki/Least_squares

$$\Delta\rho = \rho_{meas} - \rho_0 - b_u \text{ where } \rho_0^{(i)} = \left\| P^{(i)} - P_0 \right\|$$

$$\Delta\rho = G\Delta p$$

□ Unknown

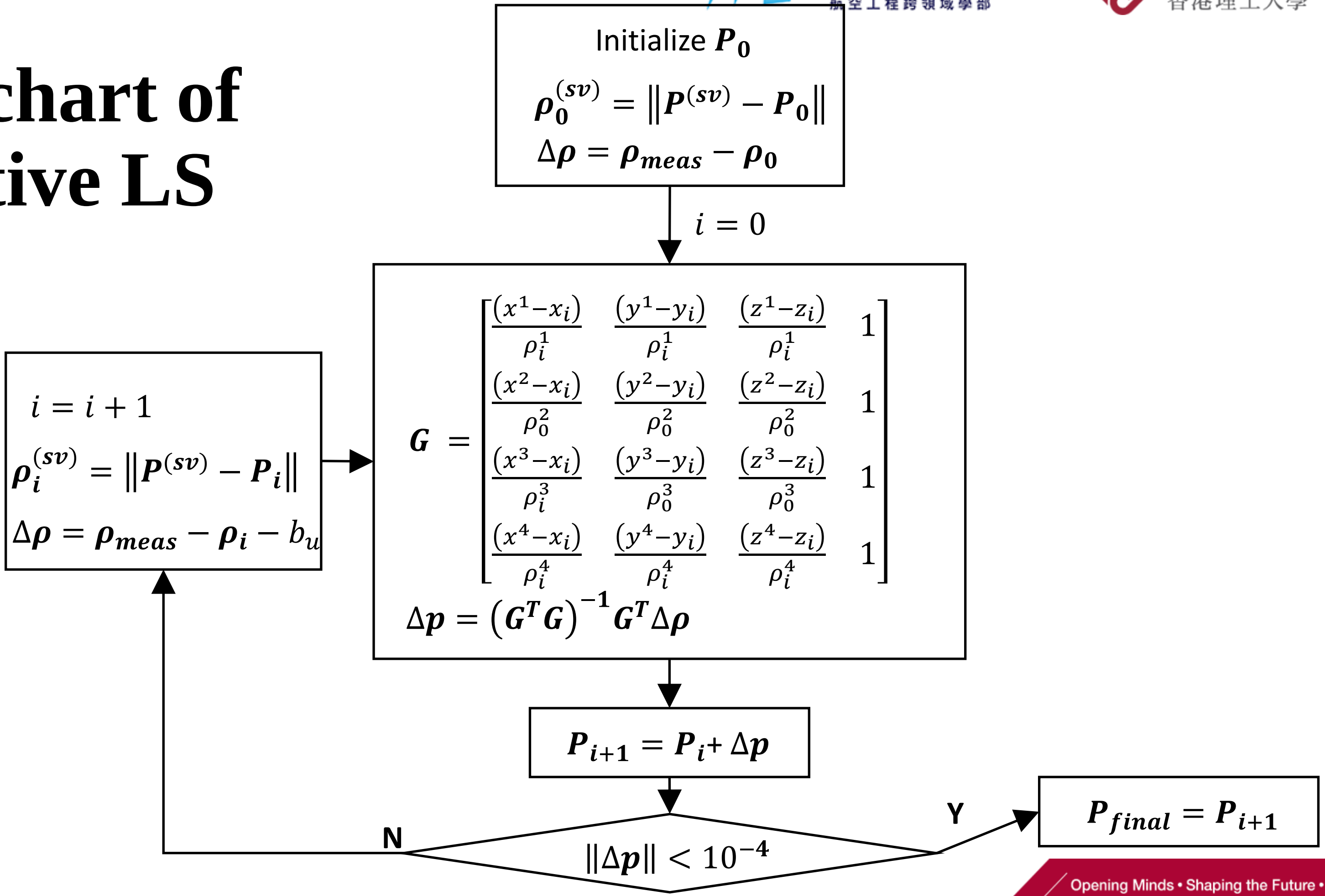
$$\begin{bmatrix} \Delta\rho^1 \\ \Delta\rho^2 \\ \Delta\rho^3 \\ \Delta\rho^4 \end{bmatrix} = \begin{bmatrix} \frac{(x^1 - x_0)}{\rho_0^1} & \frac{(y^1 - y_0)}{\rho_0^1} & \frac{(z^1 - z_0)}{\rho_0^1} & 1 \\ \frac{(x^2 - x_0)}{\rho_0^2} & \frac{(y^2 - y_0)}{\rho_0^2} & \frac{(z^2 - z_0)}{\rho_0^2} & 1 \\ \frac{(x^3 - x_0)}{\rho_0^3} & \frac{(y^3 - y_0)}{\rho_0^3} & \frac{(z^3 - z_0)}{\rho_0^3} & 1 \\ \frac{(x^4 - x_0)}{\rho_0^4} & \frac{(y^4 - y_0)}{\rho_0^4} & \frac{(z^4 - z_0)}{\rho_0^4} & 1 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ b_u \end{bmatrix}$$

$$\Delta p = G^{-1}\Delta\rho \longrightarrow \Delta p = \left(G^T G\right)^{-1} G^T \Delta\rho$$

Satellite more than 4 then we need pseudo-inverse

$$P_1 = P_0 + \Delta p$$

# Flowchart of Iterative LS



# Example of Iterations in LS method

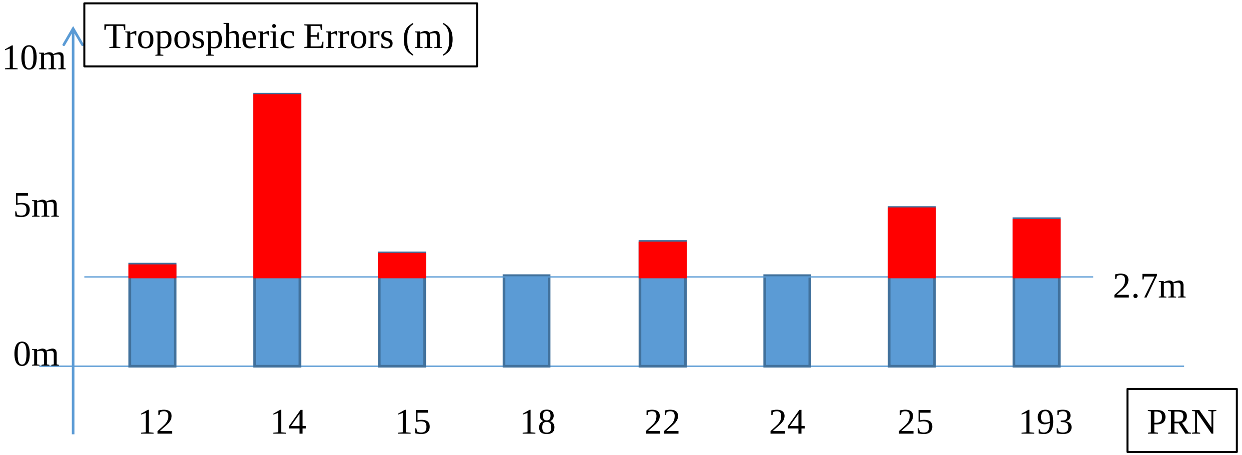
- 4 unknown variables (x,y,x,clock) are present.
- At least 4 visible satellites are required.
- With true satellite positions and true range between satellites and user antenna, the calculated position is true (only one solution).
- It is impossible in a practical sense.
- Least-Square method (LS method) is mainly used for the estimation of user antenna position.

# Example of Iterations in LS method

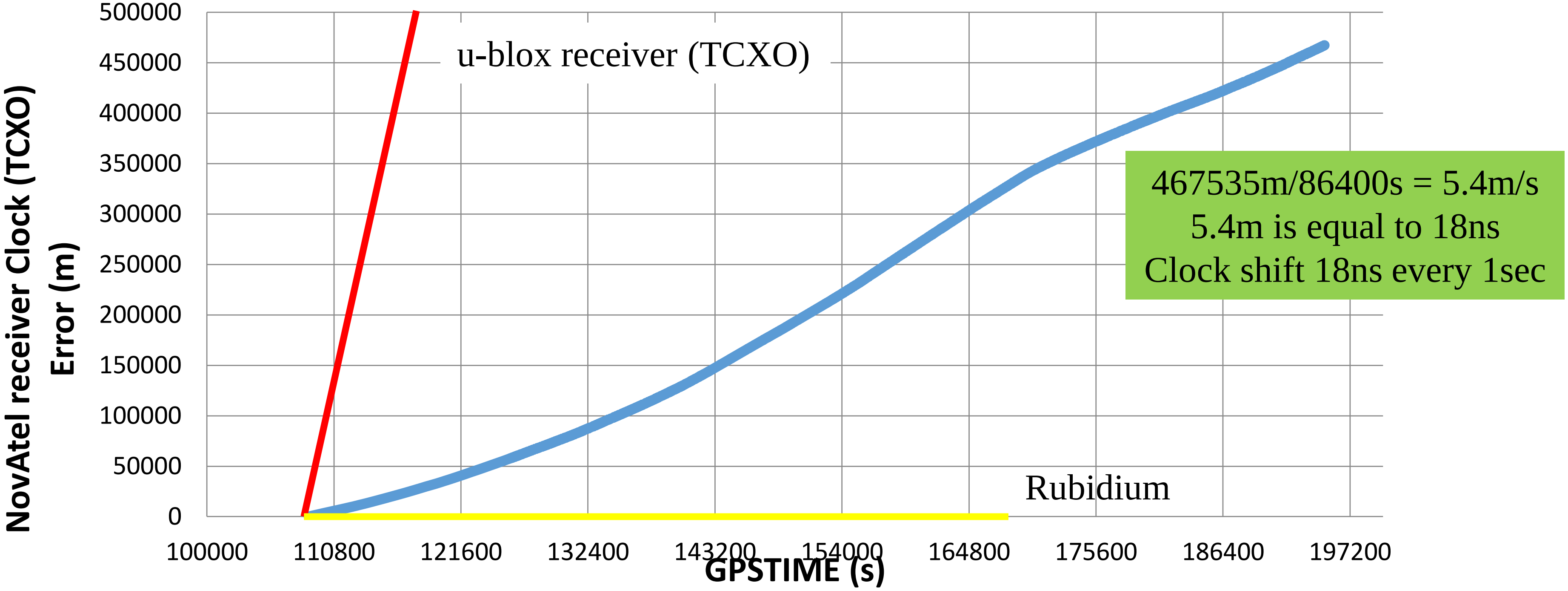
- The user antenna was located in PolyU campus.
- If we set (0, 0, 0) as an initial x, y, z positions,
- After the first iteration, the estimated position was 22.156, 114.191, 1252955m. (Po Toi Island)
- Secondly, it was 22.304, 114.101, 42298m (close to near sea of Kowloon)
- Thirdly, it was 22.305166, 114.181192, 116m (about 30m away from antenna)
- Fourth, it was  22.305843, 114.181064, 63m (within 2m from antenna)

# Common Biases are negligible

- Please remember that the common biases to all satellites are negligible in LS method. They are absorbed into clock offset term.



# What is receiver clock offset ?
# Receiver clock offset is co-product of single positioning



# Doppler Effect


One dimension is assumed. Right direction is positive.

\+

> Receiver is set in the car.

> Received frequency is

$$f_{obs} = f_s \frac{cs - v_o}{cs - v_s}$$

> "cs" is speed of light.

> Doppler frequency "$f_D$" is equal to "$f_{obs} - f_{source}$"

> FLL (frequency lock loop) tries to estimate "$f_D$".

> Once we can estimate "$f_D$", "$v_o$" can be resolved.

# Velocity Estimation

> Velocity estimation in GPS is just same as shown in the previous slide.

> The differences are as follows.

*** 3 dimension velocity ($v_{x}$, $v_{y}$, $v_{z}$) have to be estimated.**

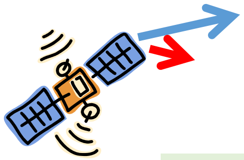
*** Frequency in the receiver is based on on-board clock.**

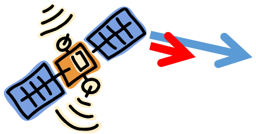
*** Measurement is pseudorange rate, which calculated from <u>Doppler frequency AND satellite velocity</u>.**

> 4 unknown variables ($v_{x}$, $v_{y}$, $v_{z}$, $f_{clk}$) have to be estimated using at least 4 visible satellites. DOP is also important.

> Velocity estimation is same as position estimation.

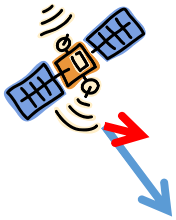
# Image of Velocity Estimation

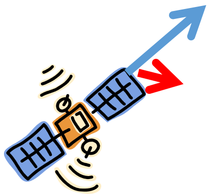


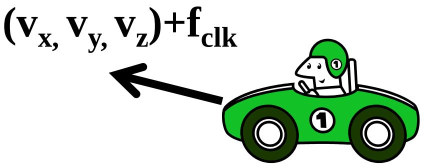


* 4 relative velocities (projection of satellite velocity on unit LOS vector to receiver) are needed to estimate car velocity (+$f_{clk}$).

* The accuracy of car velocity depends on the accuracy of satellite velocity and received frequency estimation.







# Receiver Velocity Estimation from the Doppler Measurements

Measurements from Doppler

$$\boldsymbol{y} = (-\lambda_i D_{r,i}^1, -\lambda_i D_{r,i}^2, -\lambda_i D_{r,i}^3, \ldots, -\lambda_i D_{r,i}^m)^T$$

Observation function

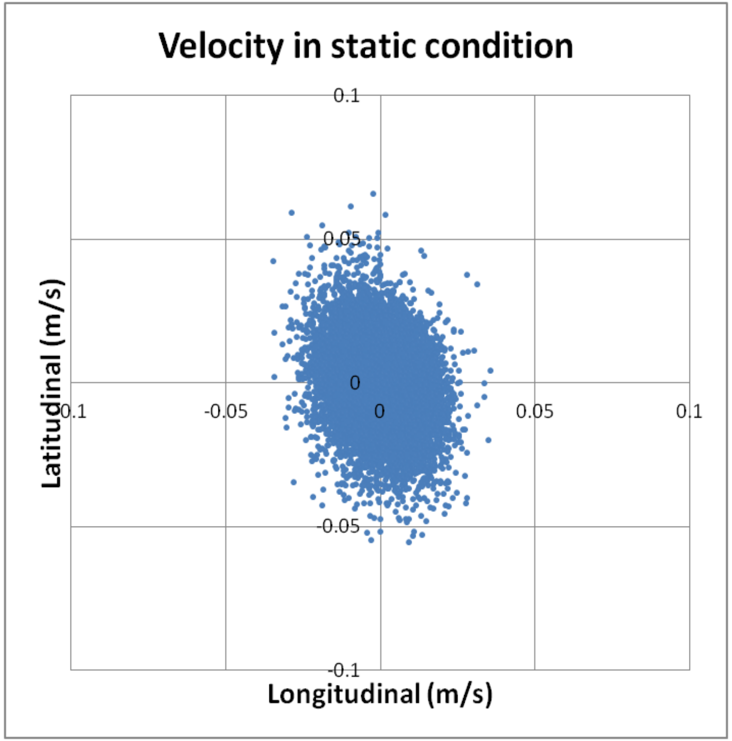
$$\boldsymbol{h}(\boldsymbol{x}) = \begin{pmatrix} \dot{r}_r^1 + c\dot{dt}_r - c\dot{dT}^1 \\ \dot{r}_r^2 + c\dot{dt}_r - c\dot{dT}^2 \\ \dot{r}_r^3 + c\dot{dt}_r - c\dot{dT}^3 \\ \vdots \\ \dot{r}_r^m + c\dot{dt}_r - c\dot{dT}^m \end{pmatrix} \quad \boldsymbol{H} = \begin{pmatrix} -\boldsymbol{e}_r^{1T} & 1 \\ -\boldsymbol{e}_r^{2T} & 1 \\ -\boldsymbol{e}_r^{3T} & 1 \\ \vdots & \vdots \\ -\boldsymbol{e}_r^{mT} & 1 \end{pmatrix} \qquad \text{(F.6.28)}$$

The range-rage $\dot{r}_r^s$ between the receiver and the satellite in these equations is derived from:

$$\dot{r}_r^s = \boldsymbol{e}_r^{sT}\left(\boldsymbol{v}^s(t^s) - \boldsymbol{v}_r\right) + \frac{\omega_e}{c}\left(v_y^s x_r + y^s v_{x,r} - v_x^s y_r - x^s v_{y,r}\right) \qquad \text{(F.6.29)}$$

Can you try to formulate the steps for GNSS velocity estimation similar to the position estimation? ☺

# Performance of GPS based Velocity




std = 1.6 cm/s

Accuracy in terms of frequency

GPS L1 wavelength = 19cm

1Hz : 19cm

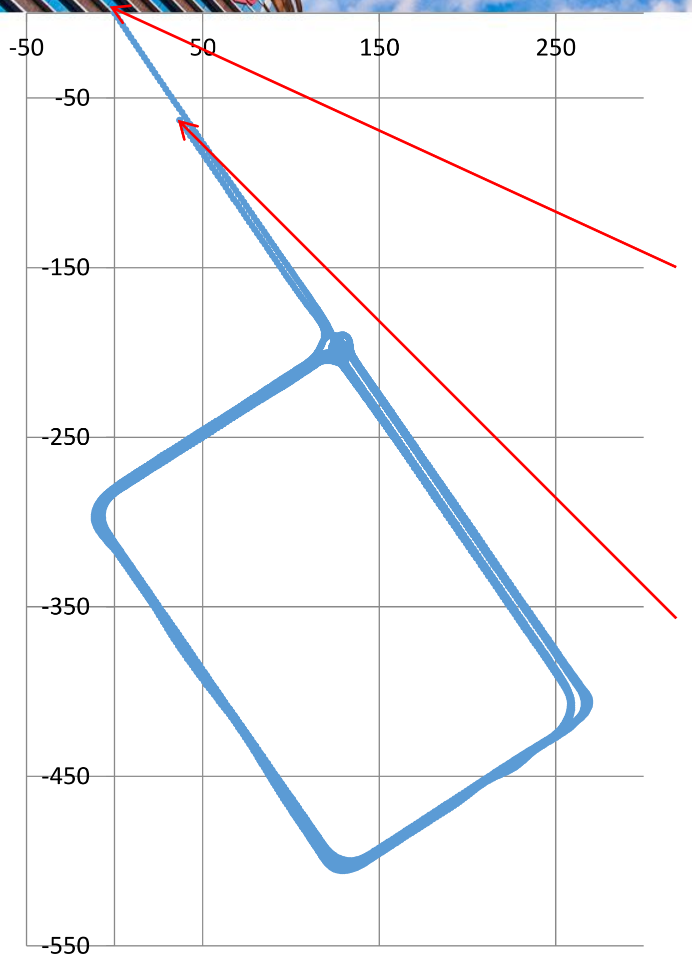
0.1Hz : 1.9cm

Accuracy in terms of satellite velocity

sv_vel [t]=(sv_vel [t+1]-sv_vel [t-1])/2

based on ephemeris parameters

Accuracy in velocity is very good comparing to accuracy in positioning.




# Moving Platform

(Sub-urban area)

- Origination : 0,0
- Velocity was accumulated.
- Data Rate : 5Hz
- Period : 650 sec
- Receiver : NovAtel OEM6
- Left and right rounds : 6 times
- End point : 36.76m,-62.91m
- RTK : 35.75m,-65.18m

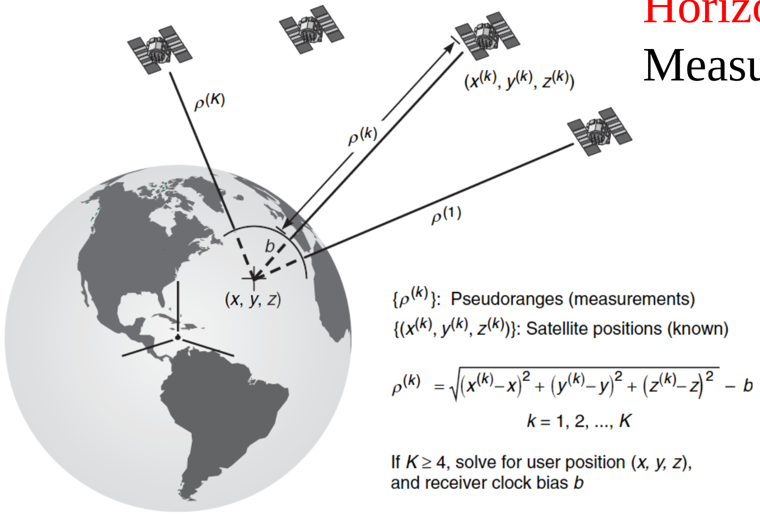
Deviation after 11 minutes velocity accumulation was about 2-3 m.

# GPS Positioning Performance

# Positioning Performance of GPS

Positioning Performance =

Measurements Accuracy × DOP




Horizontal accuracy = Measurements accuracy × HDOP

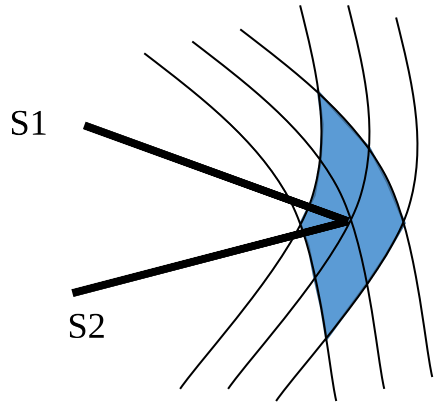
# Satellite Geometry

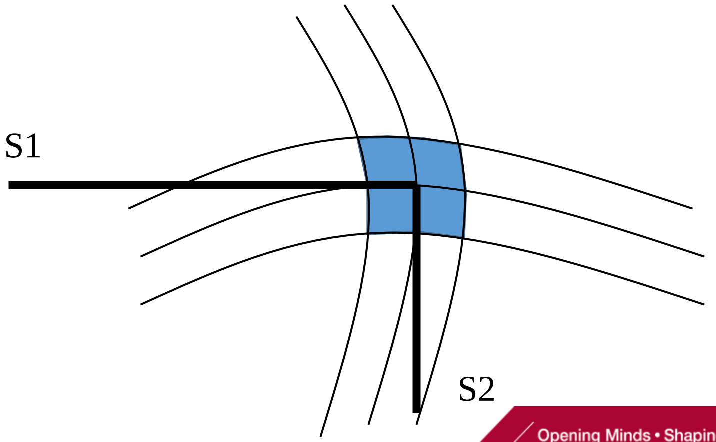
> Relative position between the user and the GPS satellites affects the accuracy of the solution
  - Geometric Dilution Of Precision GDOP
  - Position or spherical PDOP
  - Horizontal HDOP
  - Vertical VDOP
  - Time TDOP

> Lower DOP values result in better accuracy

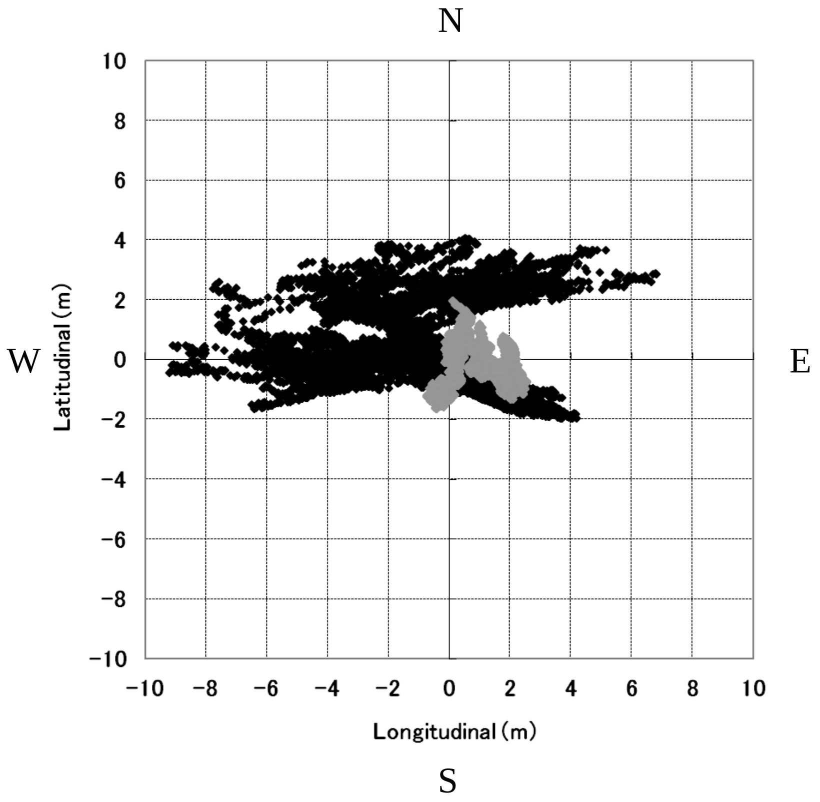
# What is DOP ?（dilution of precision : DOP）

- If the measurements errors are zero, the calculated user position is true.
- However, if the measurements include some errors, the accuracy depends on measurement errors as well as the geometry of satellites (=DOP).



# All Satellites VS. East Visible Satellites




- Only east side satellites are used in the dark color plots. (average=8.7)
- All satellites are used in the light color plots. (average=4.6)

# Calculation of DOP

$$\Delta \boldsymbol{p} = \boxed{\left(\boldsymbol{G}^T \boldsymbol{G}\right)^{-1} \boldsymbol{G}^T} \Delta \boldsymbol{\rho}$$

Mapping matrix from *pseudorange* domain to *positioning* domain

$$H = \left(G^T G\right)^{-1}$$

$$H = \begin{bmatrix} H_{11} & & & H_{ij} \\ & H_{22} & & \\ & & H_{33} & \\ H_{::} & & & H_{44} \end{bmatrix}$$

$$\sigma_e^2 = \sigma_{URE}^2 H_{11}$$

$$\sigma_n^2 = \sigma_{URE}^2 H_{22}$$

$$\sigma_u^2 = \sigma_{URE}^2 H_{33}$$

$$\sigma_b^2 = \sigma_{URE}^2 H_{44}$$

$$\text{Position DOP } (PDOP) = \sqrt{H_{11} + H_{22} + H_{33}}$$

$$\text{Geometry DOP } (GDOP) = \sqrt{H_{11} + H_{22} + H_{33} + H_{44}}$$

$$HDOP = \sqrt{H_{11} + H_{22}}$$

$$VDOP = \sqrt{H_{33}}$$

$$TDOP = \sqrt{H_{44}}$$

$$PDOP \simeq 1.5 \sim 5.0$$

$$HDOP \simeq 0.5 \sim 1.5$$

$$VDOP \simeq 1.0 \sim 4.5 \text{ (Why worse?)}$$

# Why we learn measurements and errors ?

- Needless to say, "position, velocity and time" are important for users.
- The ability to improve final performance of the above outputs strongly depends on how can we estimate or possibly mitigate measurements errors.
- Measurements errors strongly depends on the environment and receiver performance.

# Noise and Bias

- Errors are often categorized as noise and bias.

> #1 Errors in the parameter values broadcast by a satellite in its navigation message for which the Control Segment is responsible

> #2 Uncertainties associated with the propagation medium which affect the travel time of the signal from a satellite to the receiver

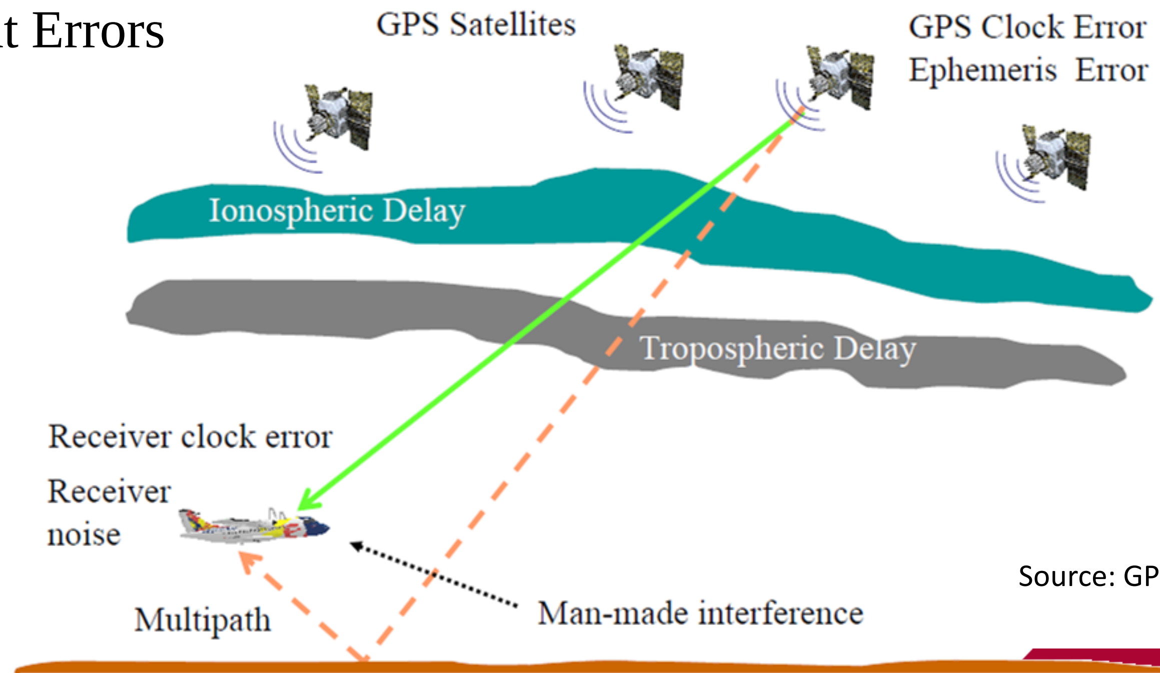
> #3 Receiver noise which affects the precision of a measurement, and interference from signals reflected from surfaces in the vicinity of the antenna

# Source of Measurements Errors

> Control Segment Errors

> Signal Propagation Modeling Errors

> Measurement Errors




Source: GPS Lab. Stanford Univ.

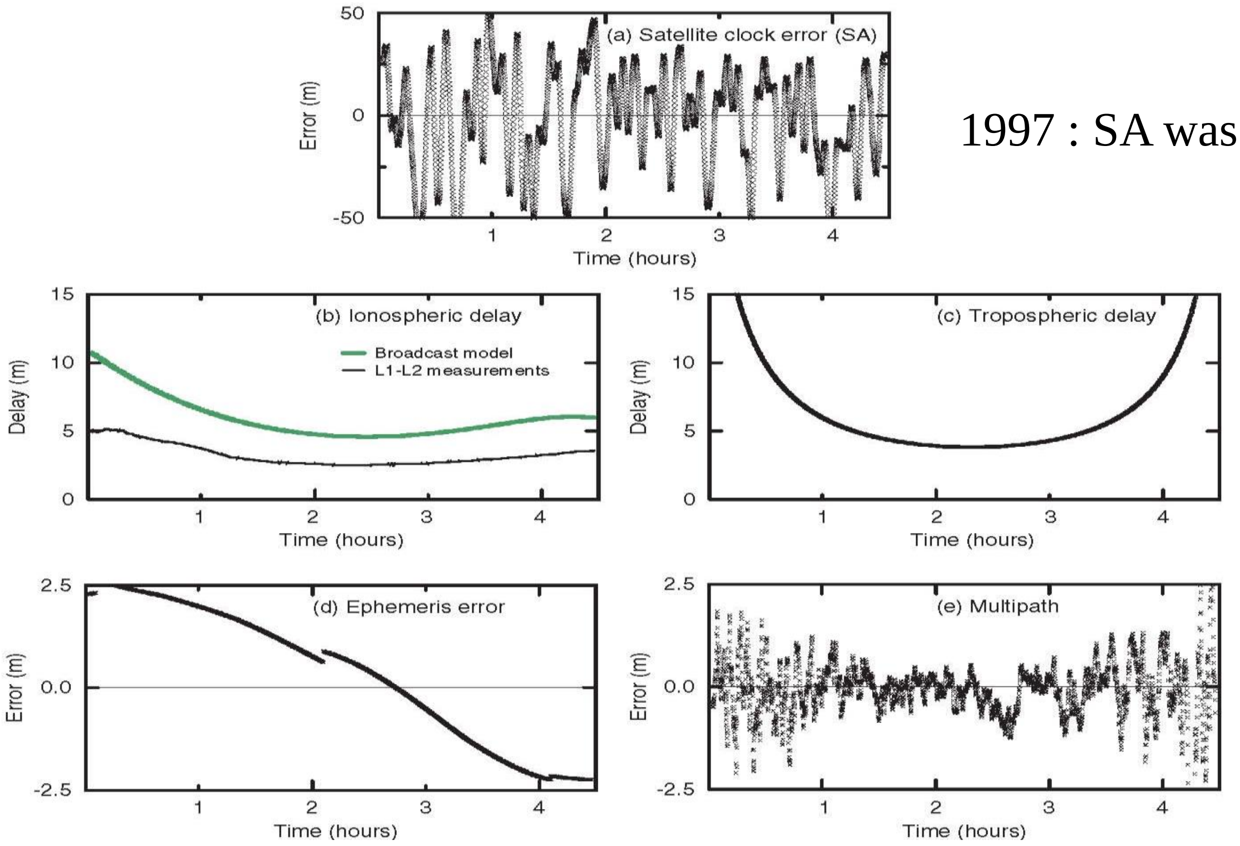
# Typical pseudo-range measurement errors for L1 receiver

| Error Source | RMS Range Error |
|---|---|
| Satellite clock and ephemeris parameters | 3 m (SIS URE) |
| Atmospheric propagation modeling | 5 m |
| Receiver noise and multipath | 1 m |
| User range error (URE) | 6 m |

Total RMS Range Error = SIS+ URE

URE : User Range Error
SIS : Signal-in-Space

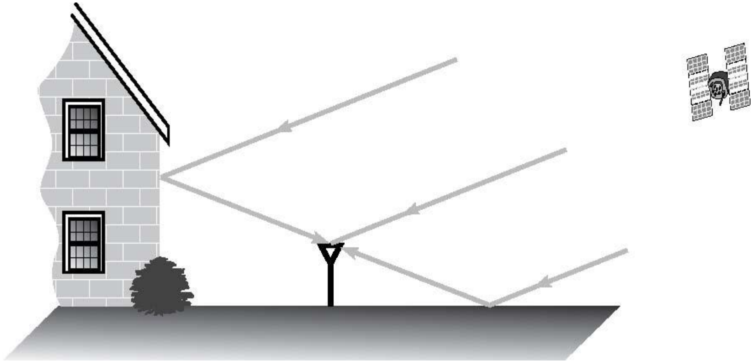
# Measurement Error : Empirical Data



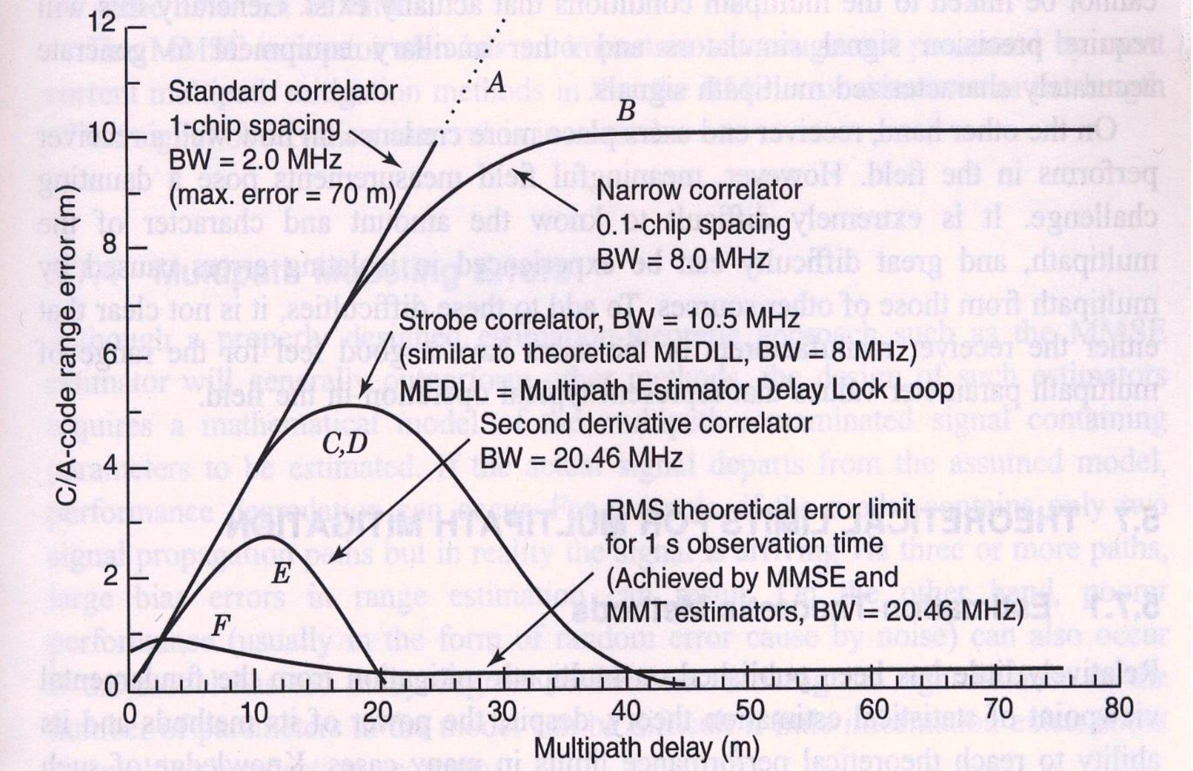

1997 : SA was activated.

# Measurement Errors - Multipath

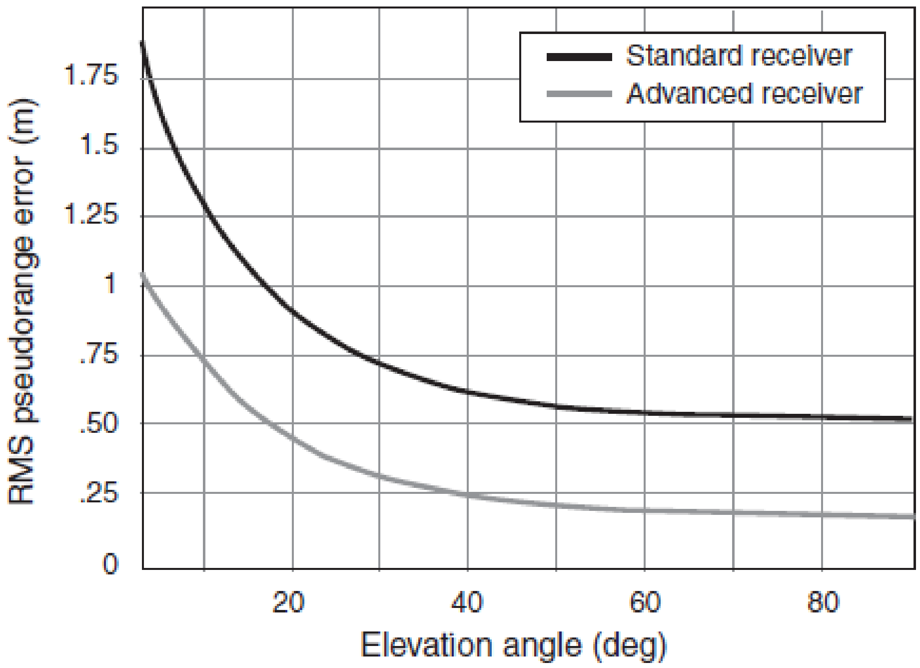
- Multipath refers to the phenomenon of a signal reaching an antenna via two or more paths.
- The range measurement error due to multipath depends on the strength of the reflected signal and the delay between direct and reflected signals.
- Mitigation of multipath errors : Antenna or Receiver

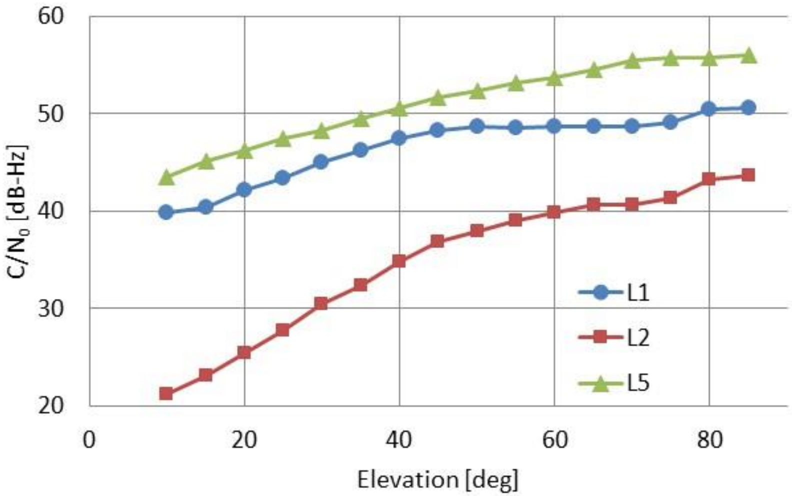


# Multipath Mitigation Technique (Receiver inside)



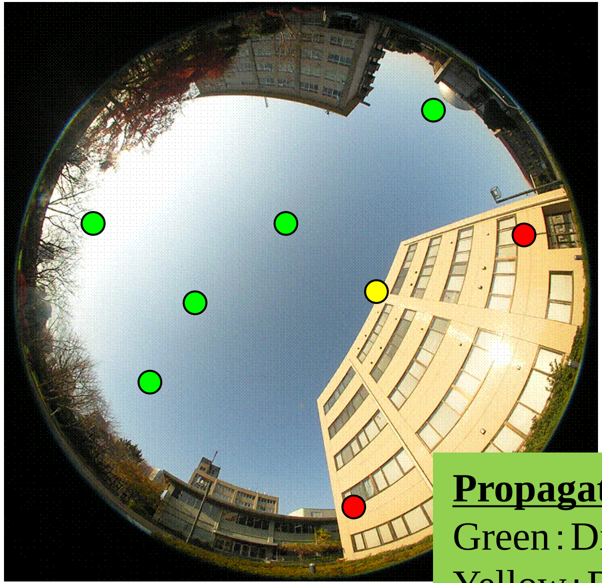
# Receiver thermal noise for two types of receiver



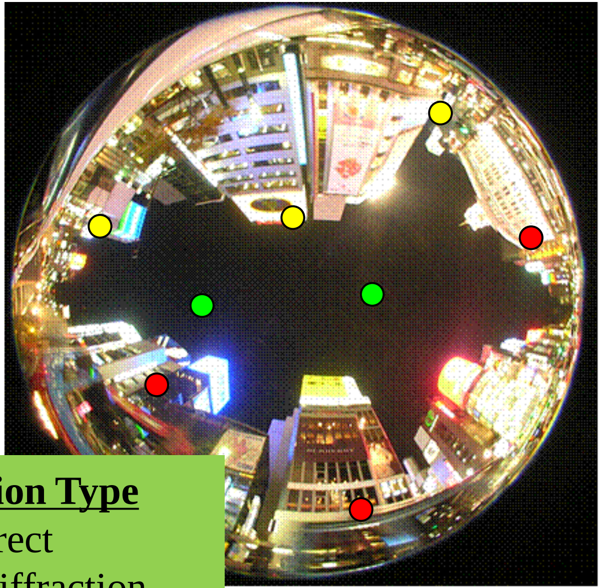




Received Signal Strength ($C/N_0$) and Elevation

# Sky Views in two different places (same constellation but different performance)



Campus

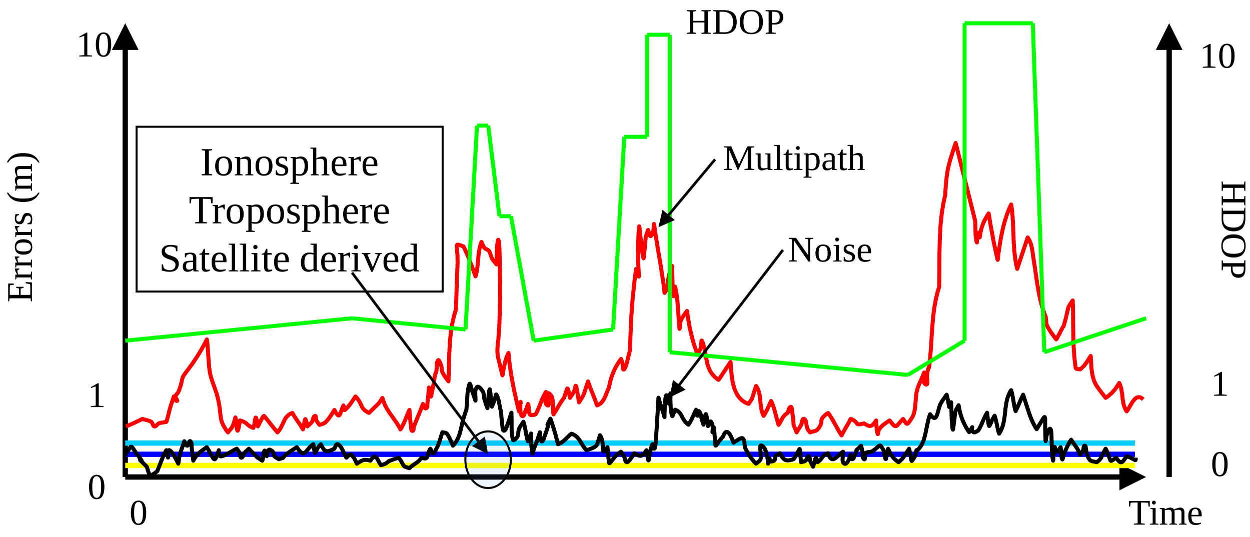


Urban Canyon

**Propagation Type**

Green : Direct
Yellow : Diffraction
Red : Masking +Reflection

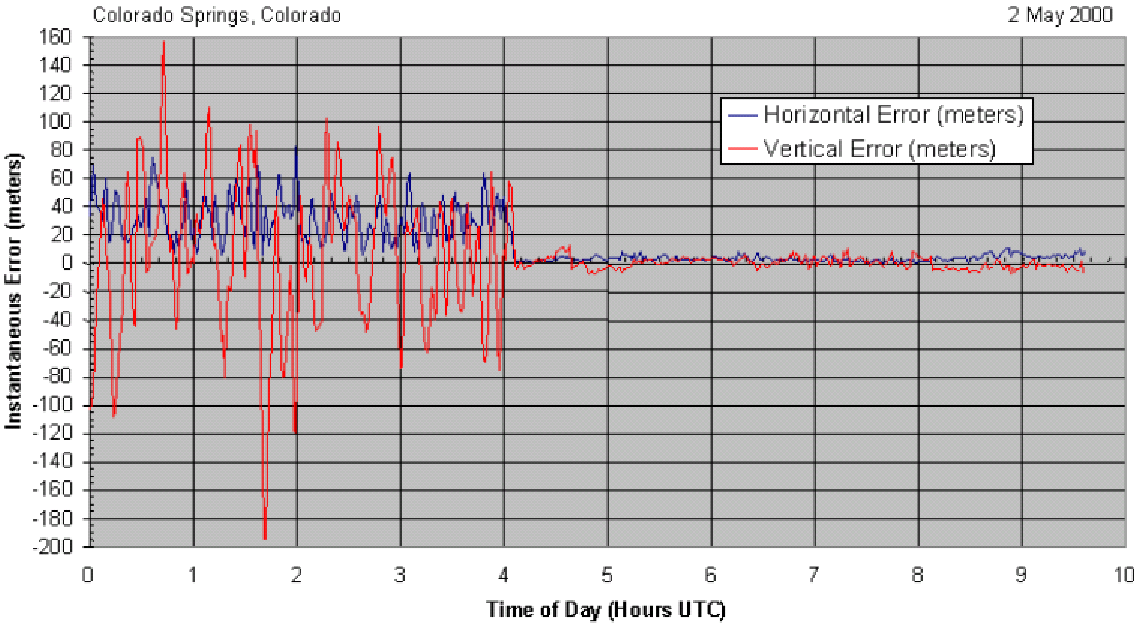
# Temporal Measurements Errors and DOP Variation (sub-urban)



# GPS Measurement Errors

| Source | Potential error size | Error mitigation using single point positioning |
|---|---|---|
| Satellite clock model | 2 m (rms) | → |
| Satellite ephemeris prediction | 2 m (rms) along the LOS | → |
| Ionospheric delay | 2-10 m (zenith)<br>Obliquity factor 3 at 5° | 1-5 m (single-freq.)<br>within 0.5m (dual-freq.) |
| Tropospheric delay | 2.3-2.5m (zenith)<br>Obliquity factor 10 at 5° | 0.1-1 m |
| Multipath (open sky) | Code : 0.5-1 m<br>Carrier : 0.5-1 cm | → |
| Receiver Noise | Code : 0.25-0.5 m (rms)<br>Carrier : 1-2 mm (rms) | → |

# History…Deactivation the artificial distortion of the signal




**On September 18, 2007, the US DoD reported that with the next generation of GPS satellites (GPS III), satellite navigation signals can no longer be artificially distorted**

# Improved Positioning

# Why we discuss about measurement errors ?

- Back to bias and noise errors discussion, noise errors of pseudo-range can be mitigated to some degree using **carrier phase smoothing technique**.

- On the other hand, you have to estimate **bias errors** as accurate as possible **by yourself** to improve positioning performance.

- All kinds of improved techniques are essentially same in terms of estimating or eliminating bias or noise errors.

# Improved GPS

> Positioning Smoothing by Carrier measurement

> DGPS (Differential GPS) and
RTK (Real Time Kinematic)
are powerful method for error mitigation.

> DGPS uses the fact that the most of error sources change slowly in the time domain if the distance between reference and user is approx. within 100km.

# Carrier Phase Measurement

$$\phi(t) = \phi_u(t) - \phi^s(t-\tau) + N$$

$$\phi(t) = f \cdot \tau + N$$

$$= \frac{r(t, t-\tau)}{\lambda} + N$$

$\varphi_u(t)$ carrier phase in the receiver

$\varphi^S(t-\tau)$ carrier pahse in the satellite

$\tau$ transit time

N integer ambiguity

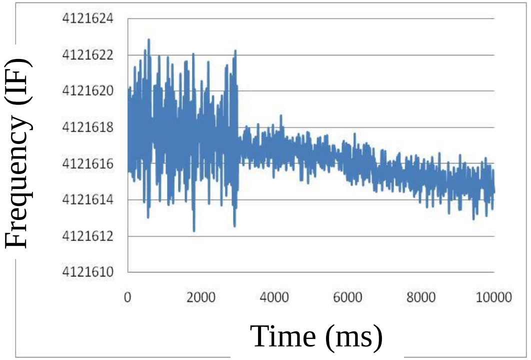
$f, \lambda$ Doppler frequency and wavelength

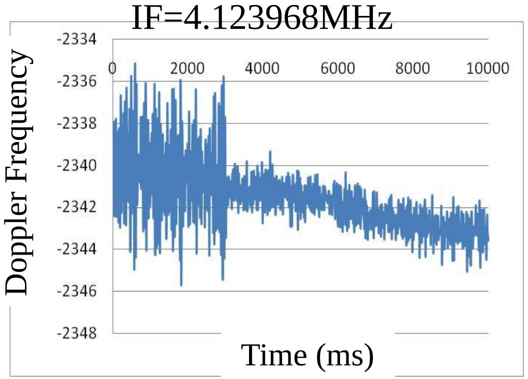
$r(t, t-\tau)$ geometrical range

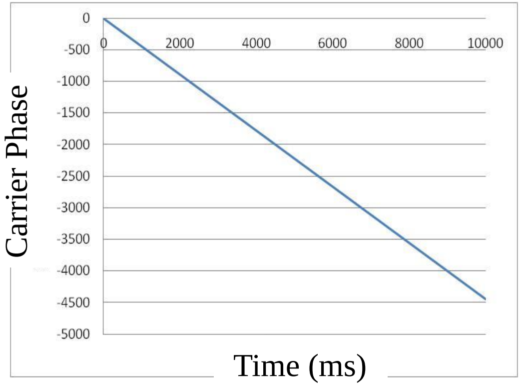
**Clock error and measurements errors are assumed zero.**

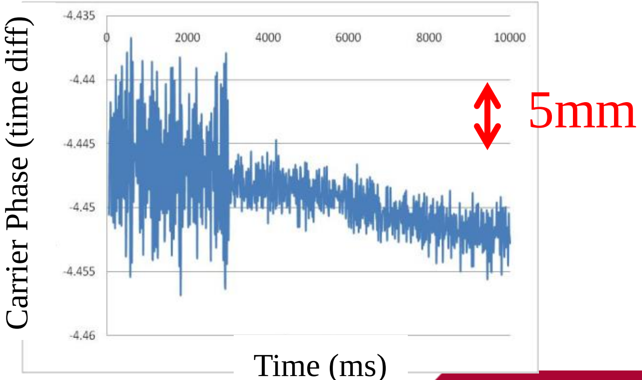
**Carrier phase measurement is accumulated Doppler frequency.**
**Be careful about "f". In the receiver, carrier frequency is basically converted to "IF".**

# Real Carrier Phase Measurements



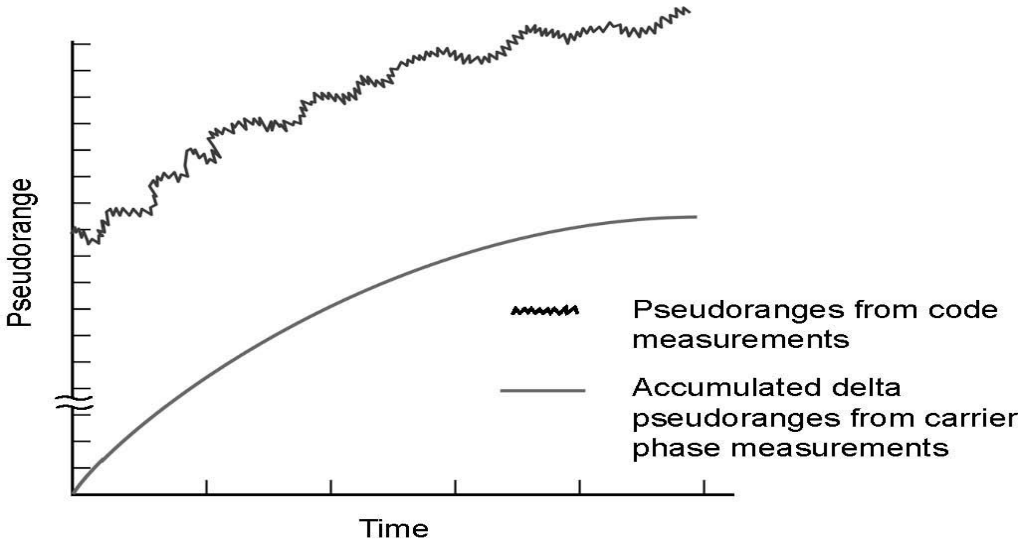



# Combining Code and Carrier Measurements

Carrier phase measurement can be used to smooth pseudo-range Measurement.




**The code-based measurements are noisy. The carrier-based estimates are precise but ambiguous, and the plot starts arbitrarily at zero value.**

# Formula of Carrier Smoothing

$$\rho^*(t) = d^{(k)}{}_u(t) + b^{(k)}{}_u(t) - B^{(k)}(t) + T(t)$$

$$\phi(t_i) = \rho^*(t_i) - I(t_i) + N\lambda$$

$$\phi(t_{i-1}) = \rho^*(t_{i-1}) - I(t_{i-1}) + N\lambda$$

$$\Rightarrow \rho^*(t_i) = \phi(t_i) - \phi(t_{i-1}) + \rho^*(t_{i-1})$$

$$\left(\text{cycles} + \frac{\text{phase}}{2048}\right) \times \lambda_{L1}$$

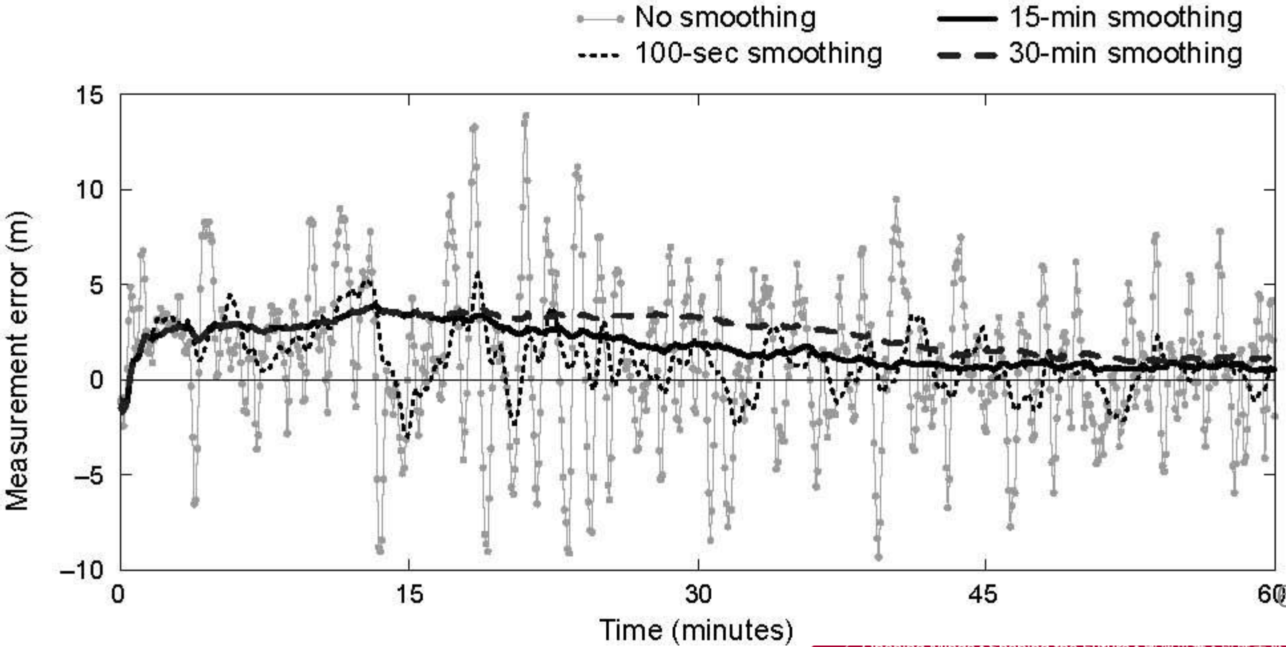
$$\overline{\rho(t_1)} = \rho(t_1)$$

$$\overline{\rho(t_{i+1})} = \frac{1}{M}\rho(t_{i+1}) + \frac{M-1}{M}\left[\overline{\rho}(t_i) + \left[\phi(t_i+1) - \phi(t_i)\right]\right]$$

# Carrier-smoothed pseudo-ranges with different filter lengths

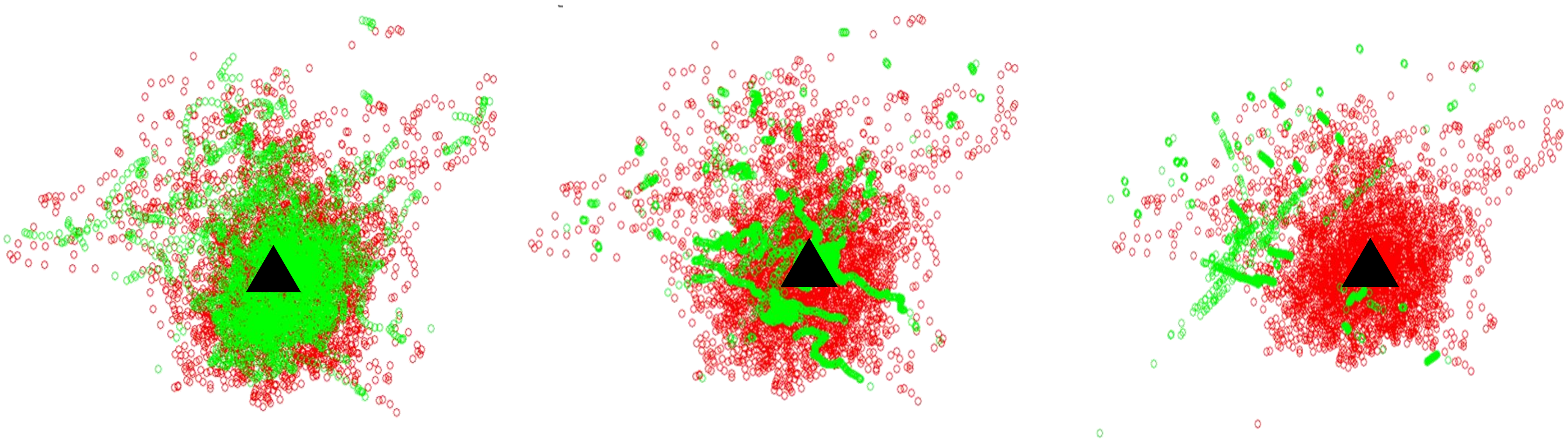
$$\bar{\rho}(t_i) = \frac{1}{M}\rho(t_i) + \frac{(M-1)}{M}[\bar{\rho}(t_{i-1}) + (\Phi(t_i) - \Phi(t_{i-1}))],$$

$$\bar{\rho}(t_1) = \rho(t_1).$$



# Carrier Smoothing Result using different M values

- M=10
- M=100
- M=5000

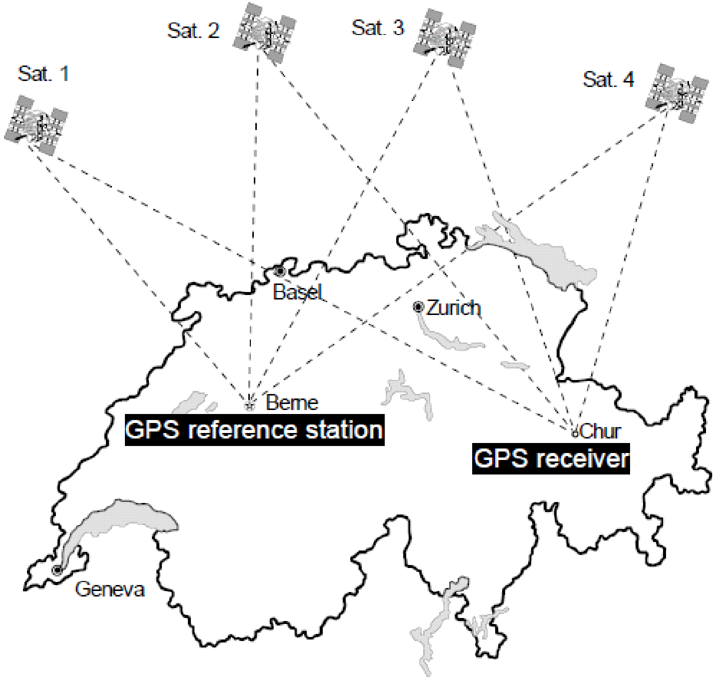


Single point positioning

Single point positioning with Carrier smoothing

True position

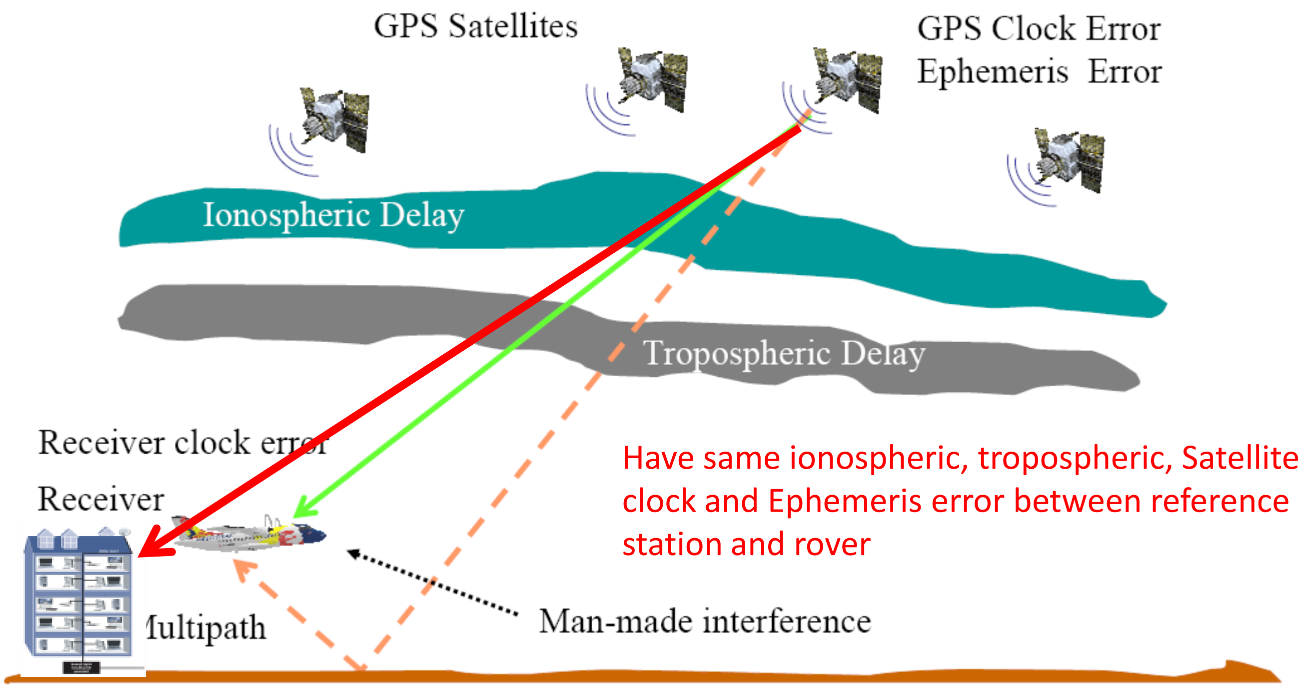
# Architecture of DGPS




- Determination of the correction values at the reference station
- Transmission of the correction values from the reference station to the GPS user
- Compensation for the determined pseudo-ranges to correct the calculated position of the GPS user

Correction [prn] = Pseudo-range[prn] – True-range [prn]

# Principle of DGPS




Have same ionospheric, tropospheric, Satellite clock and Ephemeris error between reference station and rover

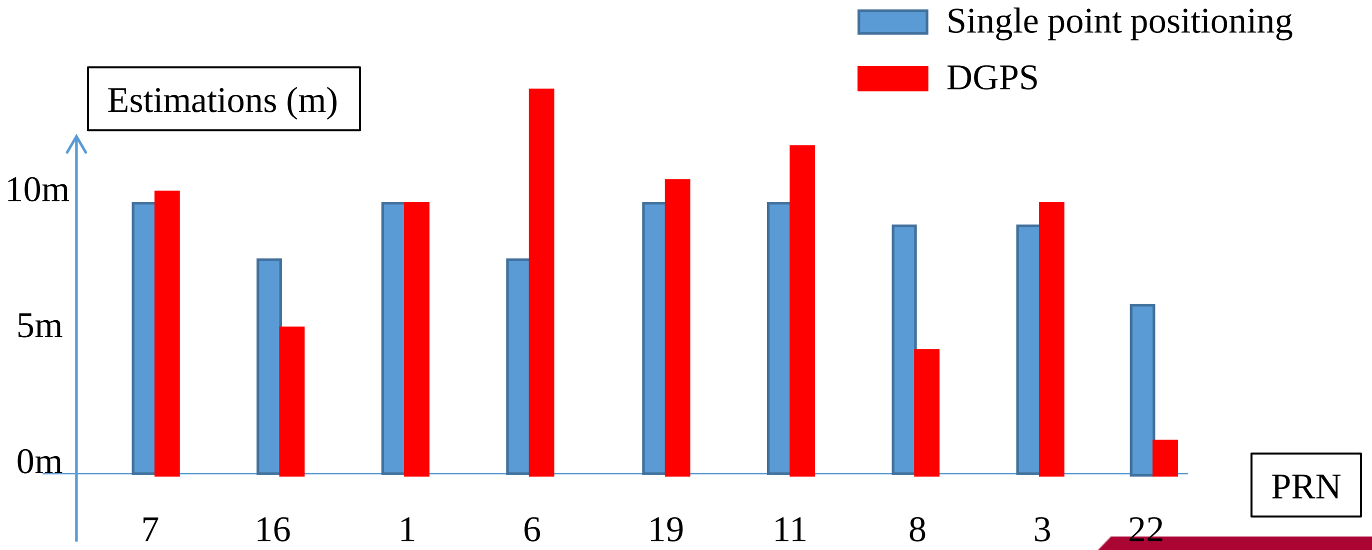
**Reference Station**

# Real Correction Data

> Correction [prn] =

Pseudo-range[prn] – True-range [prn]

> Correction data provides the better estimations in each satellite in LS method.

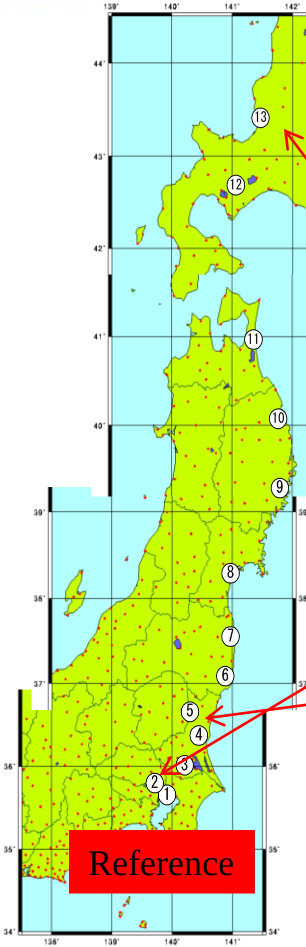


# DGPS mitigates …

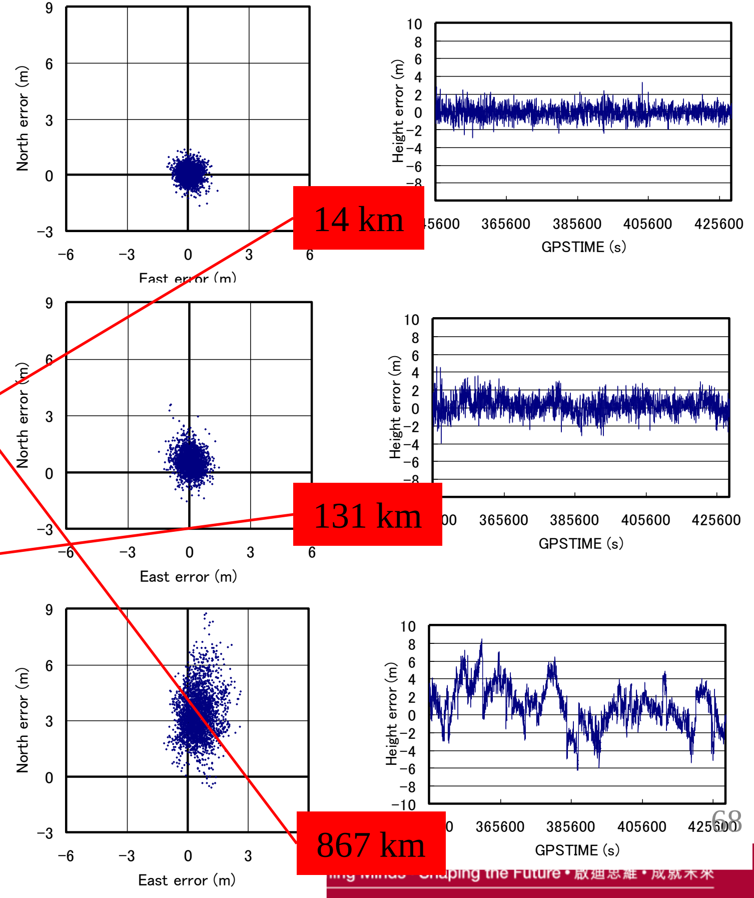
| Source | Potential error using SPP | Error mitigation using DGPS |
|---|---|---|
| Satellite clock model | 2 m (rms) | 0.0 m |
| Satellite ephemeris prediction | 2 m (rms) along the LOS | 0.1 m (rms) |
| Ionospheric delay | 2-10 m (zenith)<br>Obliquity factor 3 at 5° | 0.2 m (rms) |
| Tropospheric delay | 2.3-2.5m (zenith)<br>Obliquity factor 10 at 5° | 0.2 m (rms) + altitude effect |
| Multipath (open sky) | Code : 0.5-1 m<br>Carrier : 0.5-1 cm | Not helpful |
| Receiver Noise | Code : 0.25-0.5 m (rms)<br>Carrier : 1-2 mm (rms) | Not helpful |

rms: root mean square

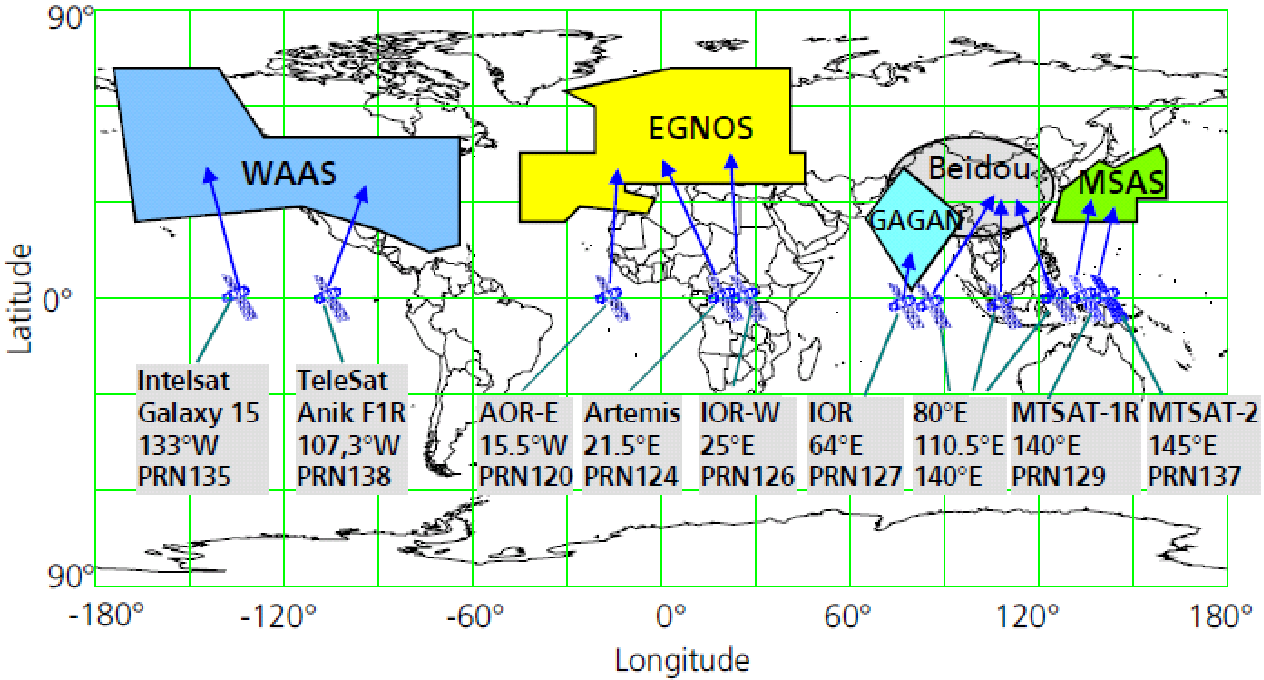
# Limitation of DGPS




| number | name | type |
|---|---|---|
| ① | 千葉市川 | 基準局 |
| ② | 足立 | 未知点 |
| ③ | 阿見 | 未知点 |
| ④ | 水戸 | 未知点 |
| ⑤ | 大田原 | 未知点 |
| ⑥ | いわき | 未知点 |
| ⑦ | 小高 | 未知点 |
| ⑧ | 利府 | 未知点 |
| ⑨ | 釜石 | 未知点 |
| ⑩ | 久慈 | 未知点 |
| ⑪ | 六ヶ所 | 未知点 |
| ⑫ | 大滝 | 未知点 |
| ⑬ | 厚田 | 未知点 |

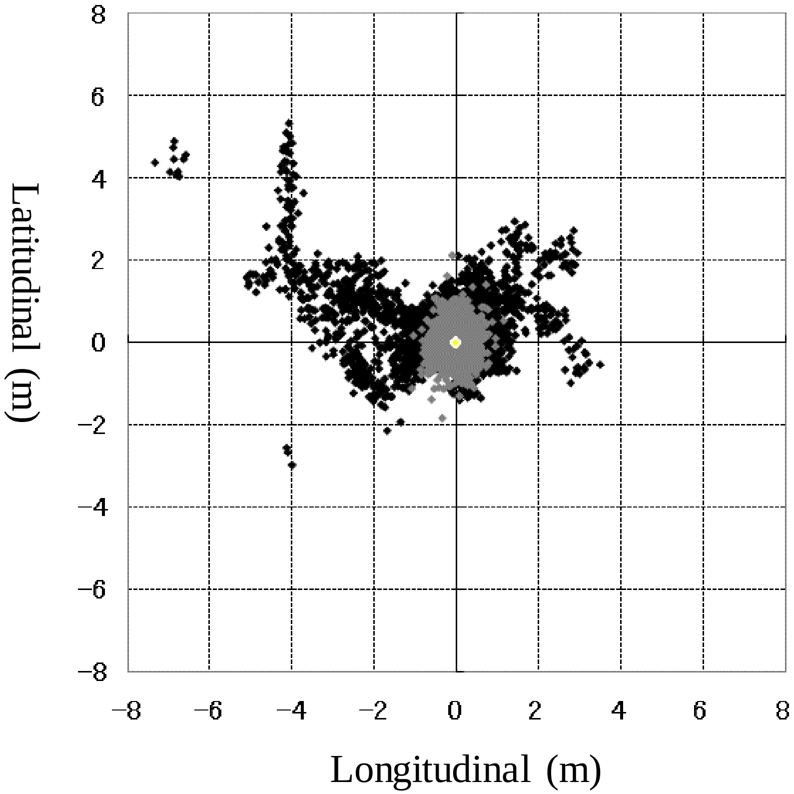


# SBAS




Without the installation of the reference stations, you can use correction data through the SBAS satellite such as MTSAT in Japan. Under quiet ionospheric condition, the performance is generally good within 1-2 m .

# DGPS and RTK Performance



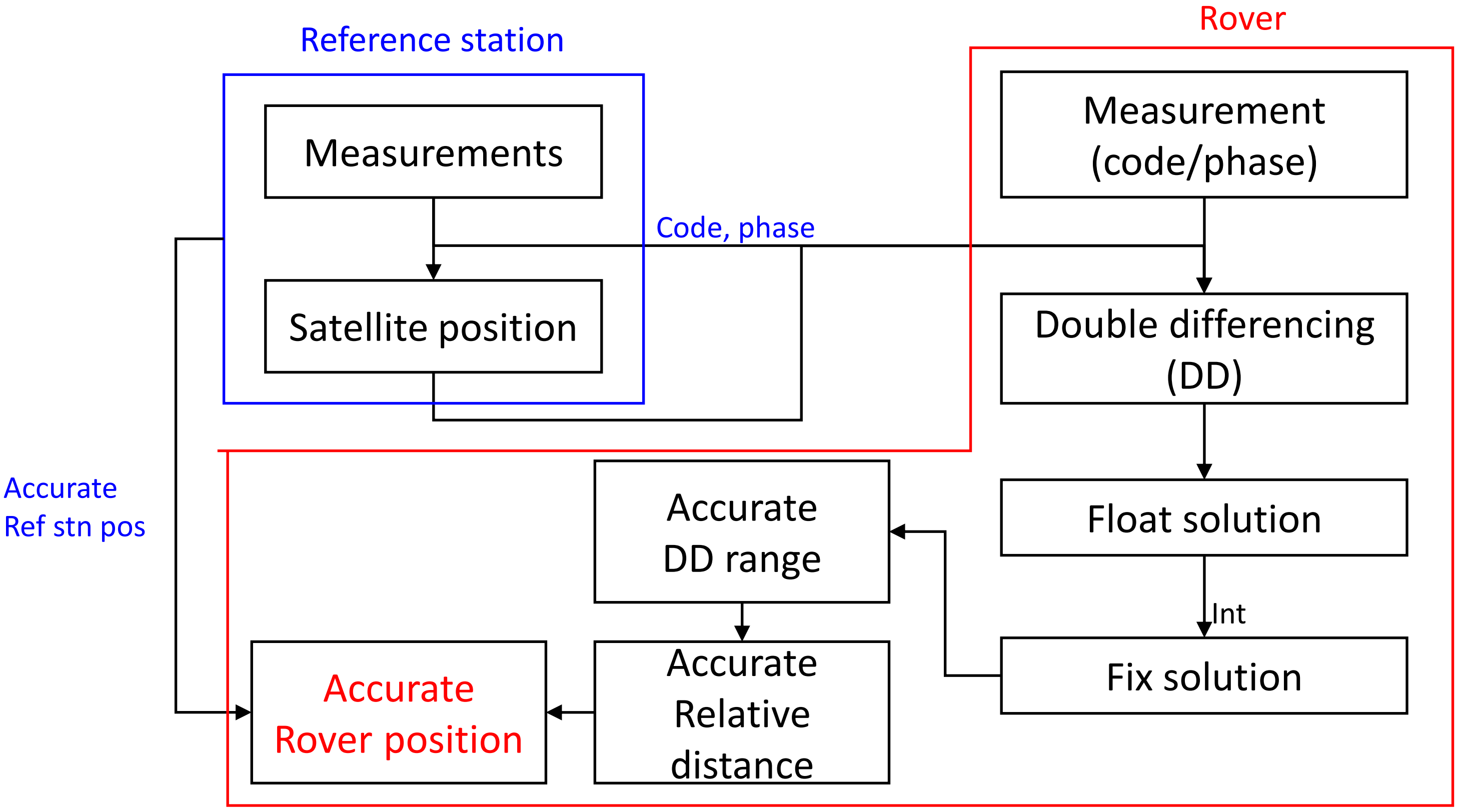

- Single Positioning
- DGPS
- RTK

Rooftop (Lab.)
15s interval
24 hours
Reference : Ichikawa

# RTK (Real Time Kinematic)

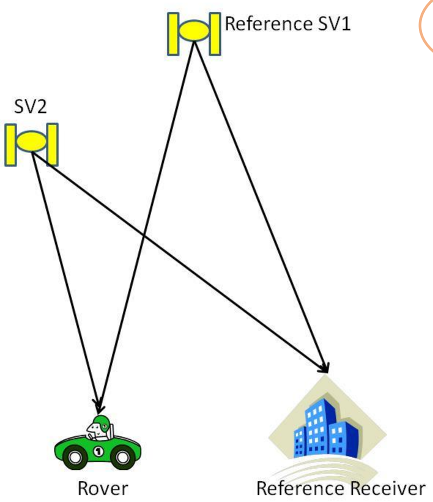
- The concept of RTK is same as DGPS.
- RTK uses carrier phase measurements. DGPS uses pseudo-range measurements.
- GPS receiver is able to measure 1/100 of wavelength of L1 frequency (19 cm).
- If you have high-end receiver, you know your position within 1-2cm accuracy as long as you have 5 or more LOS satellites.

# RTK Flowchart



# Key Concept of RTK

(double difference technique)




$$P_{rov\_ref}^{sv1\_sv2} = (P_{rov}^{sv1} - P_{ref}^{sv1}) - (P_{rov}^{sv2} - P_{ref}^{sv2})$$

$$= \rho_{rov}^{sv1} + c(dt_{sv1} - dT_{rov}) + ion_{rov}^{sv1} + tropo_{rov}^{sv1} + mp_{rov}^{sv1} + noise_{rov}^{sv1}$$
$$- \left[\rho_{ref}^{sv1} + c(dt_{sv1} - dT_{ref}) + ion_{ref}^{sv1} + tropo_{ref}^{sv1} + mp_{ref}^{sv1} + noise_{ref}^{sv1}\right]$$
$$- \left[\rho_{rov}^{sv2} + c(dt_{sv2} - dT_{rov}) + ion_{rov}^{sv2} + tropo_{rov}^{sv2} + mp_{rov}^{sv2} + noise_{rov}^{sv2}\right]$$
$$+ \left[\rho_{ref}^{sv2} + c(dt_{sv2} - dT_{ref}) + ion_{ref}^{sv2} + tropo_{ref}^{sv2} + mp_{ref}^{sv2} + noise_{ref}^{sv2}\right]$$

Completely zero — assumed zero within 10 km

$$= \rho_{rov}^{sv1} - \rho_{ref}^{sv1} + \rho_{rov}^{sv2} - \rho_{ref}^{sv2}$$
$$+ (mp_{rov}^{sv1} + noise_{rov}^{sv1}) - (mp_{ref}^{sv1} + noise_{ref}^{sv1})$$
$$- (mp_{rov}^{sv2} + noise_{rov}^{sv2}) + (mp_{ref}^{sv2} + noise_{ref}^{sv2})$$

**Generating new observation data = DD range!!!**

# Float Solution (Least Square)

$$\begin{bmatrix} \rho_n^1-\rho_n^r-(\rho_m^1-\rho_m^r) \\ \vdots \\ \rho_n^i-\rho_n^r-(\rho_m^i-\rho_m^r) \\ \Phi_n^1-\Phi_n^r-(\Phi_m^1-\Phi_m^r) \\ \vdots \\ \Phi_n^i-\Phi_n^r-(\Phi_m^i-\Phi_m^r) \end{bmatrix} = \begin{bmatrix} \frac{x^1-x_n}{R^1} & \frac{y^1-y_n}{R^1} & \frac{z^1-z_n}{R^1} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{x^i-x_n}{R^i} & \frac{y^i-y_n}{R^i} & \frac{z^i-z_n}{R^i} & 0 & \dots & 0 \\ \frac{x^1-x_n}{R^1} & \frac{y^1-y_n}{R^1} & \frac{z^1-z_n}{R^1} & \lambda^1 N^1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{y^i-y_n}{R^i} & \frac{y^i-y_n}{R^i} & \frac{z^i-z_n}{R^i} & 0 & \dots & \lambda^i N^i \end{bmatrix} \begin{bmatrix} \Delta x_n \\ \Delta y_n \\ \Delta z_n \\ \Delta N^1 \\ \vdots \\ \Delta N^i \end{bmatrix}$$

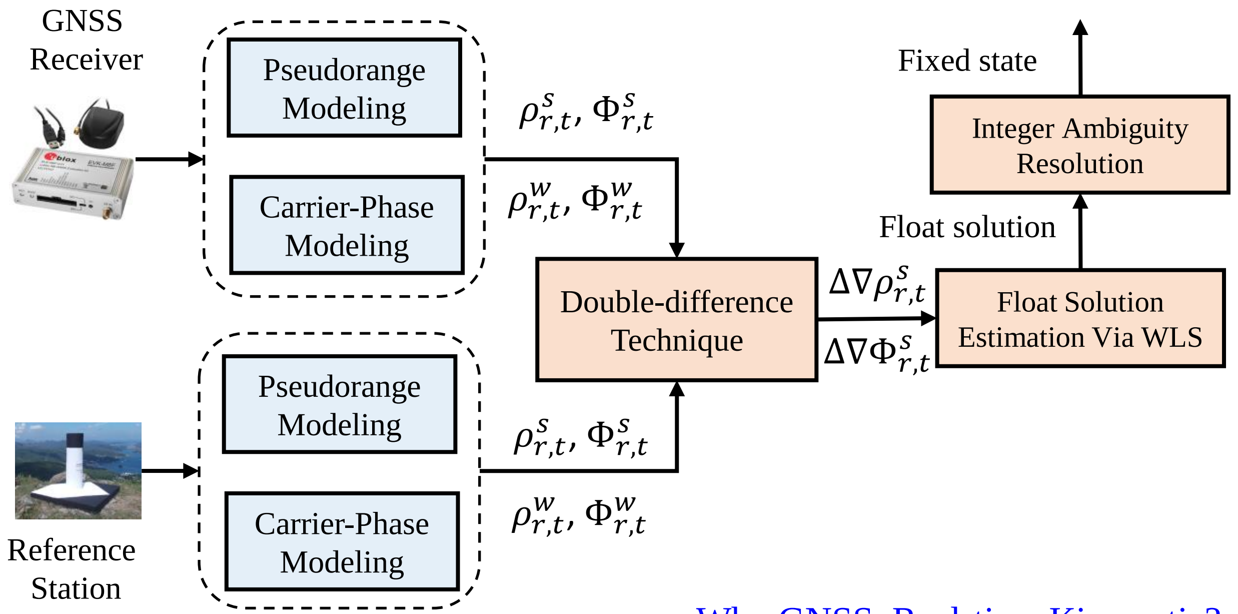
Double difference range (both pseudorange and carrierphase)

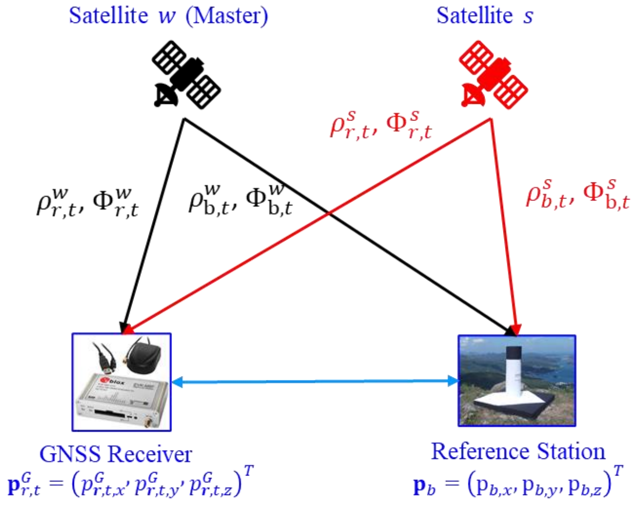
Float solution of the relative position between rover and ref stn

float ambiguity

# Overview of GNSS Real-time Kinematic

WLS*: Weighted Least Squares
DD *: Double-difference




$\rho^s_{r,t}$: Pseudorange measurement
$\Phi^s_{r,t}$: Carrier-phase measurement
$\Delta\nabla\rho^s_{r,t}$: DD Pseudorange measurement
$\Delta\nabla\Phi^s_{r,t}$: DD Carrier-phase measurement

Why GNSS Real-time Kinematic?

- Remove the error from receiver/satellite clock bias, atmosphere error using double-difference technique.
- Use the high-accuracy carrier-phase measurements.

# Observation Model for Pseudorange/Carrier Measurements

## Observation function for pseudorange (code) measurement

$$\rho_{r,t}^{s} = r_{r,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) + I_{r,t}^{s} + T_{r,t}^{s} + \varepsilon_{r,t}^{s}$$

- $\rho_{r,t}^{s}$: Pseudorange
- $r_{r,t}^{s}$: Range distance, $||\mathbf{p}_{t}^{G,s} - \mathbf{p}_{r,t}^{G}||$
- $\delta_{r,t}$: Receiver clock Bias (1~2m)
- $\delta_{r,t}^{s}$: Satellite clock bias
- $I_{r,t}^{s}$: ionospheric delay Distance (1~2m)
- $T_{r,t}^{s}$: tropospheric delay Distance (1~2m)
- $\varepsilon_{r,t}^{s}$: multipath effects, NLOS receptions, receiver noise, antenna phase-related noise (0~100m)

## Observation function for carrier-phase measurement

$$\psi_{r,t}^{s} = r_{r,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) + I_{r,t}^{s} + T_{r,t}^{s} + \varepsilon_{r,t}^{s} + N_{r,t}^{s}$$

- $\psi_{r,t}^{s}$: Carrier-phase range
- $r_{r,t}^{s}$: Range distance, $||\mathbf{p}_{t}^{G,s} - \mathbf{p}_{r,t}^{G}||$
- $\delta_{r,t}$: Receiver clock Bias (1~2m)
- $\delta_{r,t}^{s}$: Satellite clock bias
- $I_{r,t}^{s}$: ionospheric delay Distance (1~2m)
- $T_{r,t}^{s}$: tropospheric delay Distance (1~2m)
- $\varepsilon_{r,t}^{s}$: multipath effects, NLOS receptions, receiver noise, antenna phase-related noise (0~100m)
- $N_{r,t}^{s}$: Ambiguity

To use the carrier-phase measurements, the ambiguity need to be resolved.

# Single Difference Pseudorange Measurements

Single difference between the GNSS receiver and the reference station to remove the atmosphere errors:

$$\rho_{r,t}^{s} = r_{r,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) + I_{r,t}^{s} + T_{r,t}^{s} + \varepsilon_{r,t}^{s}$$

$$\rho_{b,t}^{s} = r_{b,t}^{s} + c\left(\delta_{b,t} - \delta_{b,t}^{s}\right) + I_{b,t}^{s} + T_{b,t}^{s} + \varepsilon_{b,t}^{s}$$

$$\Delta\rho_{r,t}^{s} = \rho_{r,t}^{s} - \rho_{b,t}^{s} = r_{r,t}^{s} - r_{b,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) - c\left(\delta_{b,t} - \delta_{b,t}^{s}\right)$$

Satellite $s$

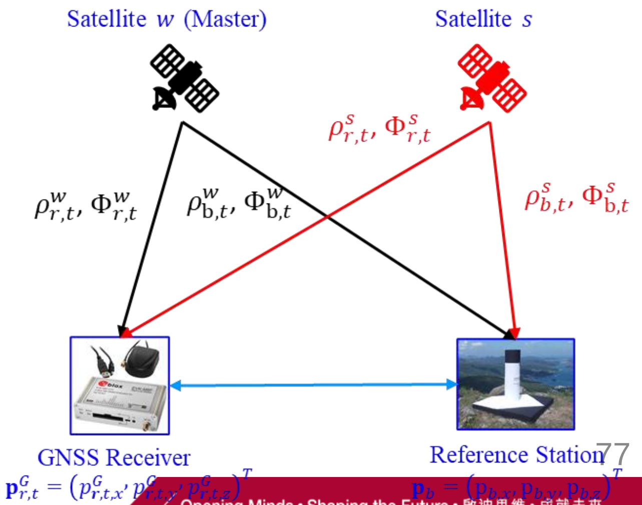
$$\rho_{r,t}^{w} = r_{r,t}^{w} + c\left(\delta_{r,t} - \delta_{r,t}^{w}\right) + I_{r,t}^{w} + T_{r,t}^{w} + \varepsilon_{r,t}^{w}$$

$$\rho_{b,t}^{w} = r_{b,t}^{w} + c\left(\delta_{b,t} - \delta_{b,t}^{w}\right) + I_{b,t}^{w} + T_{b,t}^{w} + \varepsilon_{b,t}^{w}$$

$$\Delta\rho_{r,t}^{w} = \rho_{r,t}^{w} - \rho_{b,t}^{w} = r_{r,t}^{w} - r_{b,t}^{w} + c\left(\delta_{r,t} - \delta_{r,t}^{w}\right) - c\left(\delta_{b,t} - \delta_{b,t}^{w}\right)$$

Satellite $w$ (Master)

**Assumption**: GNSS receiver and the reference station are close with the same atmosphere errors



# Double Difference Pseudorange Measurements

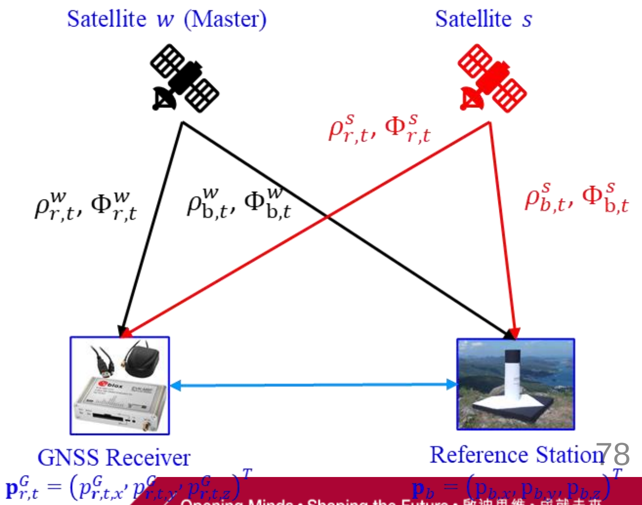
Second difference between the master satellite and the satellite $s$ to remove the atmosphere errors:

**Assumption**: GNSS receiver and the reference station are close with the same atmosphere errors

$$\Delta\rho^{s}_{r,t} = \rho^{s}_{r,t} - \rho^{s}_{b,t} = r^{s}_{r,t} - r^{s}_{b,t} + c\left(\delta_{r,t} - \delta^{s}_{r,t}\right) - c\left(\delta_{b,t} - \delta^{s}_{b,t}\right)$$ Satellite $s$

$$\Delta\rho^{w}_{r,t} = \rho^{w}_{r,t} - \rho^{w}_{b,t} = r^{w}_{r,t} - r^{w}_{b,t} + c\left(\delta_{r,t} - \delta^{w}_{r,t}\right) - c\left(\delta_{b,t} - \delta^{w}_{b,t}\right)$$ Satellite $w$ (Master)

$$\Delta\nabla\rho^{s}_{r,t} = \Delta\rho^{s}_{r,t} - \Delta\rho^{w}_{r,t} = \rho^{s}_{r,t} - \rho^{s}_{b,t} - \rho^{w}_{r,t} - \rho^{w}_{b,t}$$ DD measurements



# Single Difference Carrier-phase Measurements

Single difference between the GNSS receiver and the reference station to remove the atmosphere errors:

**Assumption**: GNSS receiver and the reference station are close with the same atmosphere errors

$$\psi_{r,t}^{s} = r_{r,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) + I_{r,t}^{s} + T_{r,t}^{s} + \varepsilon_{r,t}^{s} + N_{r,t}^{s}$$

$$\psi_{b,t}^{s} = r_{b,t}^{s} + c\left(\delta_{b,t} - \delta_{b,t}^{s}\right) + I_{b,t}^{s} + T_{b,t}^{s} + \varepsilon_{b,t}^{s} + N_{b,t}^{s}$$

Satellite $s$

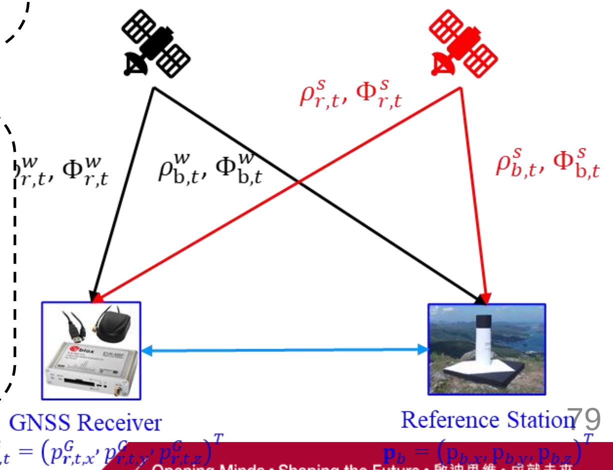
$$\Delta\psi_{r,t}^{s} = \psi_{r,t}^{s} - \psi_{b,t}^{s} = r_{r,t}^{s} - r_{b,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) - c\left(\delta_{b,t} - \delta_{b,t}^{s}\right) + N_{r,t}^{s} - N_{b,t}^{s}$$

$$\psi_{r,t}^{w} = r_{r,t}^{w} + c\left(\delta_{r,t} - \delta_{r,t}^{w}\right) + I_{r,t}^{w} + T_{r,t}^{w} + \varepsilon_{r,t}^{w} + N_{r,t}^{w}$$

$$\psi_{b,t}^{w} = r_{b,t}^{w} + c\left(\delta_{b,t} - \delta_{b,t}^{w}\right) + I_{b,t}^{w} + T_{b,t}^{w} + \varepsilon_{b,t}^{w} + N_{b,t}^{w}$$

Satellite $w$

$$\Delta\psi_{r,t}^{w} = \psi_{r,t}^{w} - \psi_{b,t}^{w} = r_{r,t}^{w} - r_{b,t}^{w} + c\left(\delta_{r,t} - \delta_{r,t}^{w}\right) - c\left(\delta_{b,t} - \delta_{b,t}^{w}\right) + N_{r,t}^{w} - N_{b,t}^{w}$$



# Double Difference Pseudorange Measurements

Second difference between the master satellite and the satellite $s$ to remove the atmosphere errors:

**Assumption**: GNSS receiver and the reference station are close with the same atmosphere errors

Satellite $s$

$$\Delta\psi_{r,t}^{s} = \psi_{r,t}^{s} - \psi_{b,t}^{s} = r_{r,t}^{s} - r_{b,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) - c\left(\delta_{b,t} - \delta_{b,t}^{s}\right) + N_{r,t}^{s} - N_{b,t}^{s}$$

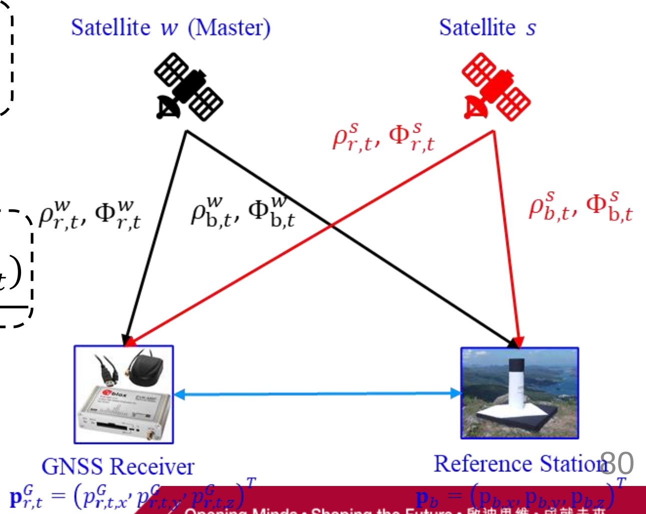
Satellite $w$ (Master)

$$\Delta\psi_{r,t}^{w} = \psi_{r,t}^{w} - \psi_{b,t}^{w} = r_{r,t}^{w} - r_{b,t}^{w} + c\left(\delta_{r,t} - \delta_{r,t}^{w}\right) - c\left(\delta_{b,t} - \delta_{b,t}^{w}\right) + N_{r,t}^{w} - N_{b,t}^{w}$$

DD measurements

$$\Delta\nabla\psi_{r,t}^{s} = \Delta\psi_{r,t}^{s} - \Delta\psi_{r,t}^{w} = \rho_{r,t}^{s} - \rho_{b,t}^{s} - \rho_{r,t}^{w} - \rho_{b,t}^{w} + \underline{\left(N_{r,t}^{s} - N_{b,t}^{s}\right) - \left(N_{r,t}^{w} - N_{b,t}^{w}\right)}$$

DD Ambiguity: $\Delta\nabla N_{r,t}^{s}$



# GNSS Real-time Kinematic Positioning: Float Solution Estimation

Estimate the float solution via weighted least square positioning

$$\begin{bmatrix} \mathbf{p}_{r,t}^{G} \\ \Delta\nabla N_{r,t}^{1} \\ \Delta\nabla N_{r,t}^{2} \\ \cdots \\ \Delta\nabla N_{r,t}^{m-1} \end{bmatrix} = \left({\mathbf{G}_t^G}^T \mathbf{W}_t \mathbf{G}_t^G\right)^{-1} {\mathbf{G}_t^G}^T \mathbf{W}_t \begin{bmatrix} \Delta\nabla \rho_{r,t}^{1} \\ \Delta\nabla \rho_{r,t}^{2} \\ \vdots \\ \Delta\nabla \rho_{r,t}^{m-1} \\ \Delta\nabla \psi_{r,t}^{1} \\ \Delta\nabla \psi_{r,t}^{2} \\ \cdots \\ \Delta\nabla \psi_{r,t}^{m-1} \end{bmatrix}$$

Output ⟹

$\mathbf{p}_{r,t}^{G}$ : Float solution of position of GNSS receiver

$\Delta\nabla N_{r,t}^{1}, \Delta\nabla N_{r,t}^{2}, \ldots$: Float ambiguity

$\left({\mathbf{G}_t^G}^T \mathbf{W}_t \mathbf{G}_t^G\right)^{-1}$ : Covariance matrix

$\mathbf{p}_{r,t}^{G}$ : Position of GNSS receiver

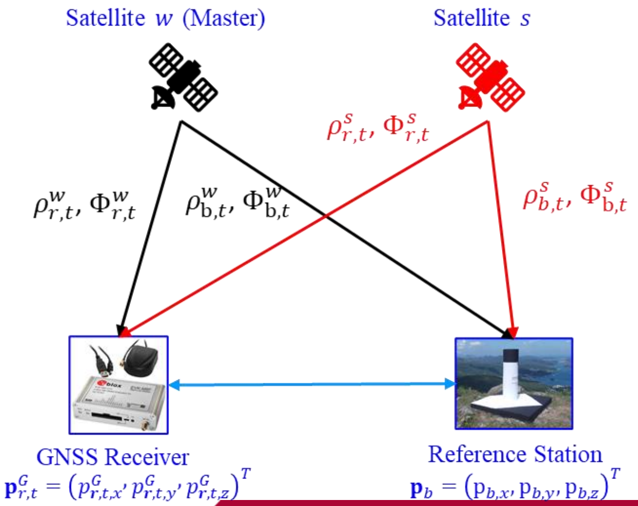
$\mathbf{W}_t$: Weighting matrix

$m$: number of satellite

$\mathbf{G}_t^G$: Observation matrix

$\Delta\nabla \rho_{r,t}^{m-1}$: DD pseudorange measurements

$\Delta\nabla \psi_{r,t}^{m-1}$: DD carrier-phase measurements



# GNSS Real-time Kinematic Positioning: Ambiguity Resolution

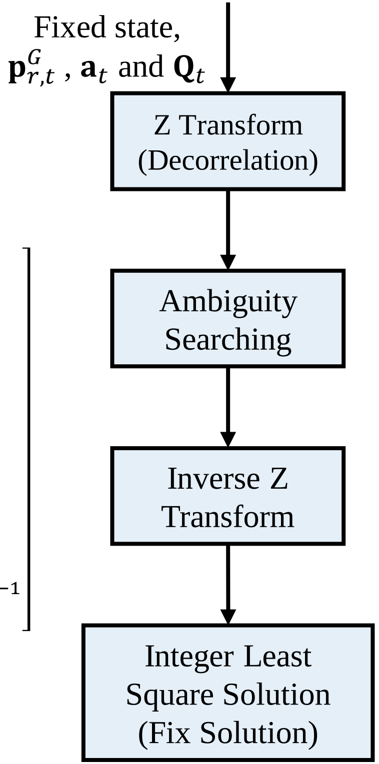
$\mathbf{p}_{r,t}^{G}$ : Float solution of position of GNSS receiver

$\mathbf{a}_t = \Delta\nabla N_{r,t}^{1}, \Delta\nabla N_{r,t}^{2}, \ldots$: Float ambiguity

$\mathbf{Q}_t = \left({\mathbf{G}_t^G}^T \mathbf{W}_t \mathbf{G}_t^G\right)^{-1}$ : Covariance matrix

$$\mathbf{G}_t^G = \begin{bmatrix} \frac{p_{t,x}^{G,1} - p_{r,t,x}^{G}}{\|\mathbf{p}_t^{G,1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,y}^{G,1} - p_{r,t,y}^{G}}{\|\mathbf{p}_t^{G,1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,z}^{G,1} - p_{r,t,z}^{G}}{\|\mathbf{p}_t^{G,1} - \mathbf{p}_{r,t}^{G}\|} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{p_{t,x}^{G,m-1} - p_{r,t,x}^{G}}{\|\mathbf{p}_t^{G,m-1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,y}^{G,m-1} - p_{r,t,y}^{G}}{\|\mathbf{p}_t^{G,m-1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,z}^{G,m-1} - p_{r,t,z}^{G}}{\|\mathbf{p}_t^{G,m-1} - \mathbf{p}_{r,t}^{G}\|} & 0 & \cdots & 0 \\ \frac{p_{t,x}^{G,1} - p_{r,t,x}^{G}}{\|\mathbf{p}_t^{G,1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,y}^{G,1} - p_{r,t,y}^{G}}{\|\mathbf{p}_t^{G,1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,z}^{G,1} - p_{r,t,z}^{G}}{\|\mathbf{p}_t^{G,1} - \mathbf{p}_{r,t}^{G}\|} & \Delta\nabla N_{r,t}^{1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{p_{t,x}^{G,m-1} - p_{r,t,x}^{G}}{\|\mathbf{p}_t^{G,m-1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,y}^{G,m-1} - p_{r,t,y}^{G}}{\|\mathbf{p}_t^{G,m-1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,z}^{G,m-1} - p_{r,t,z}^{G}}{\|\mathbf{p}_t^{G,m-1} - \mathbf{p}_{r,t}^{G}\|} & 0 & \cdots & \Delta\nabla N_{r,t}^{m-1} \end{bmatrix}$$

The correct fixed solution relies on the accuracy of float solution $\mathbf{p}_{r,t}^{G}$ and the covariance matrix $\mathbf{Q}_t$.




$\hat{\mathbf{z}}_t = \mathbf{Z}^T \mathbf{a}_t \qquad \mathbf{Q}_{t,z} = \mathbf{Z}^T \mathbf{Q}_t \mathbf{Z}$

Larger covariance matrix $\mathbf{Q}_t$, larger searching space

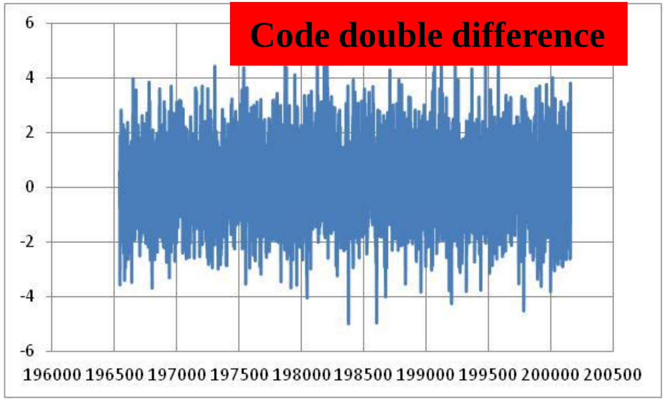
$\breve{\mathbf{z}}_t = \min_{z \in Z^{m-1}} \|\hat{\mathbf{z}}_t - \mathbf{Z}_t\|_{\mathbf{Q}_{t,z}}^2$

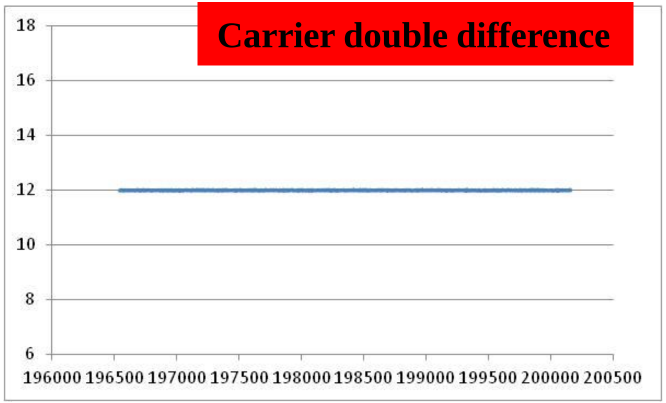
$\breve{\mathbf{a}}_t = \mathbf{Z}^{-T} \breve{\mathbf{z}}_t$

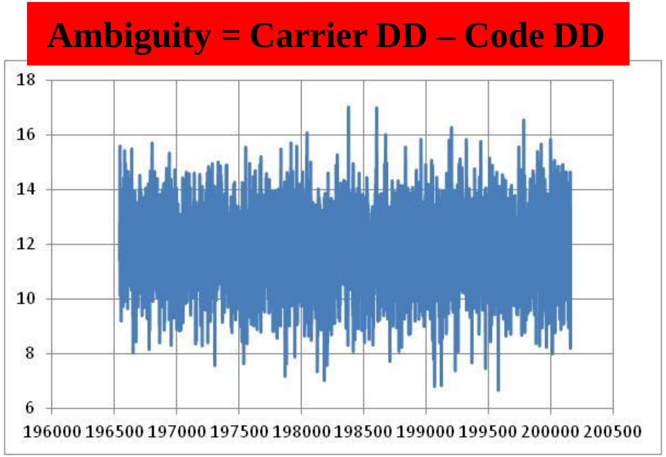
$\mathbf{p}_{r,t}^{G} = \mathbf{p}_{r,t}^{G} - \mathbf{Q}_{t,pa} {\mathbf{Q}_{t,a}}^{-1} (\mathbf{a}_t - \breve{\mathbf{a}}_t)$

# Double Differenced Observation

(open sky condition : prn19->prn3 : 1 hour)

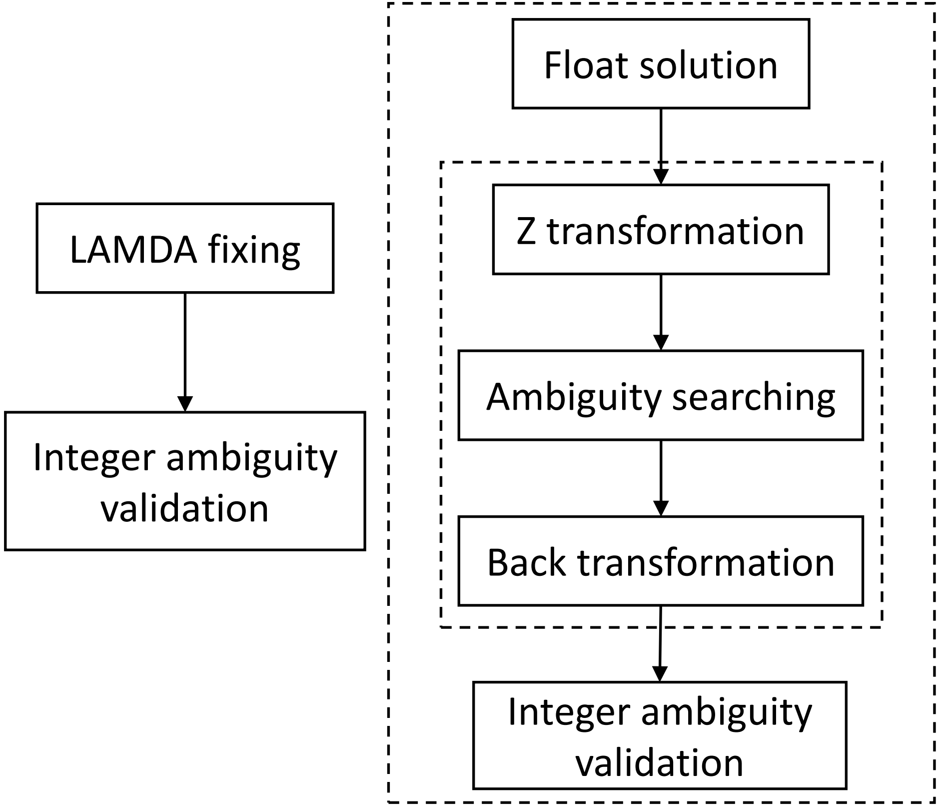



**Average = 11.8**
**Std = 1.4**

# Ambiguity Resolution by LAMBDA

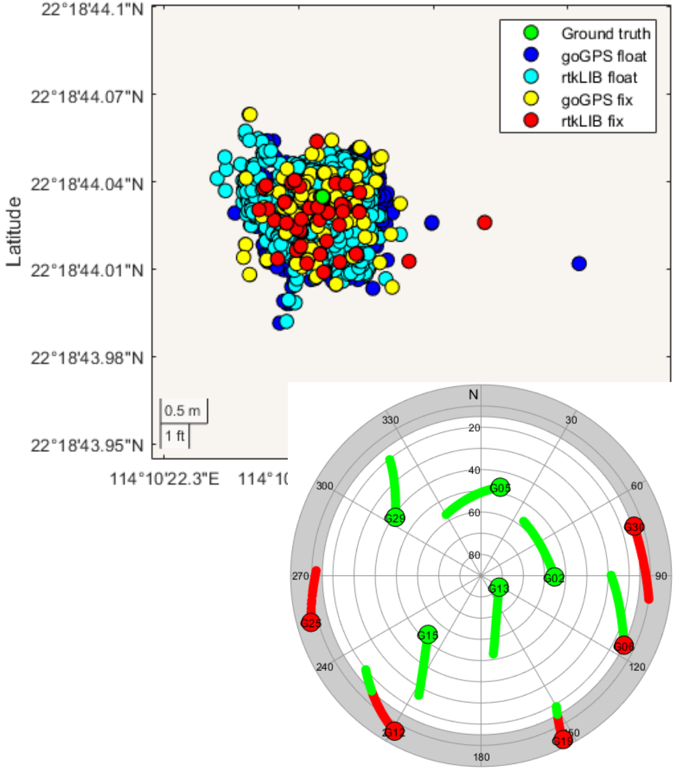



Integer ambiguity validation

$Q_{\hat{a}}$: covariance matrix of ambiguity
$\hat{a}$: estimate ambiguity float
$\breve{a}_u, \breve{a}_l$: upper & lower bound of integer ambiguity

Nominator component $O_1 = (\hat{a} - \breve{a}_l)Q_{\hat{a}}^{-1}(\hat{a} - \breve{a}_l)$
Denominator component $O_2 = (\hat{a} - \breve{a}_u)Q_{\hat{a}}^{-1}(\hat{a} - \breve{a}_u)$
Ratio test $O = {}^{O_2}/_{O_1} > 3$

# Test RTK Fix




Receiver: ublox F9P
Constellation: GPS
Frequency: L1 only
Epochs: 3562 (00:59:22); 3533 available
Elev. mask: 15°; SNR mask: 15

| Method | Fixing Rate (%) | RMS 2D Error (m) | | |
|---|---|---|---|---|
| | | Fix | Float | All |
| goGPS | 15.34 | 0.476 | 0.396 | 0.409 |
| RTKLIB (2.4.3 b33) | 20.10 | 0.197 | 0.386 | 0.356 |
| WLS | 100 | 0.805 | | |


# Multi-GNSS RTK Test using Car

| Test | Schedule |
|---|---|
| 1st | 2014/8/13 13:07–13:32 |
| 2nd | 2014/8/13 17:26–17:52 |
| 3rd | 2014/8/13 22:26–22:50 |
| 4th | 2014/8/14 8:36–9:02 |
| 5th | 2014/8/14 12:07–12:35 |

* GPS/QZS/GLONASS/GALILEO/BeiDou are entirely used in this test
* Trimble SPS855 receiver was used
* RTK : Trimble and Laboratory engine

# Summary of Test Results

Multi-GNSS RTK

| | Average NUS | Fix rate |
|---|---|---|
| Test 1 | 12.3 | 58.7% |
| Test 2 | 12.3 | 75.4% |
| Test 3 | 13.6 | 65.5% |
| Test 4 | 12.4 | 60.0% |
| Test 5 | 14.2 | 70.5% |

GPS VS. Multi-GNSS RTK
(using two same receivers : SPS855)

| Test 5 | Average NUS | Fix rate |
|---|---|---|
| GPS | 5.8 | 26.8% |
| Multi-GNSS | 14.2 | 70.5% |

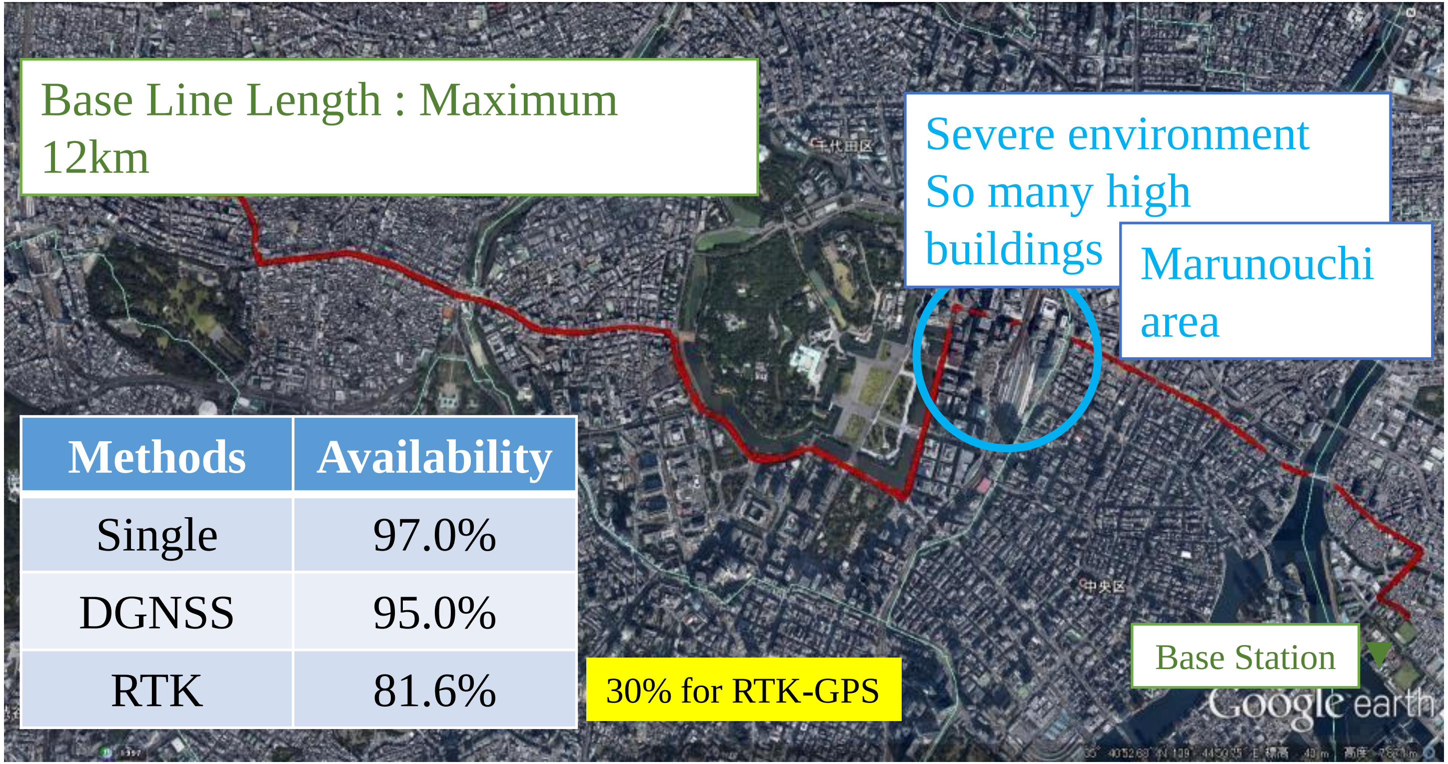
Velocity : Doppler based velocity output
G:GPS J:QZSS C:BeiDou R:GLONASS

FIX rate comparison between GNSS combinations

| Test 3 | G | GJ | GC | GR | GJC | GJCR |
|---|---|---|---|---|---|---|
| RTK FIX rate (%) | 48.2 | 58.2 | 55.5 | 55.4 | 64.7 | 65.9 |

The reason for small contribution of BeiDou/GLONASS to RTK was just due to the shortage of high elevation those satellites

# Height Determination using Automobile



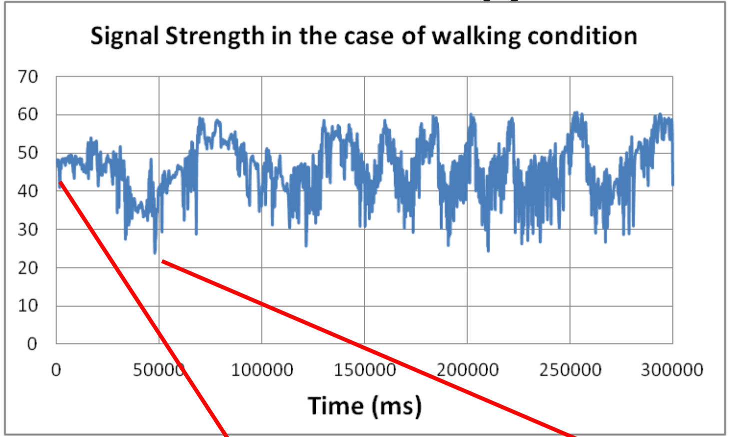

| Methods | Availability |
|---|---|
| Single | 97.0% |
| DGNSS | 95.0% |
| RTK | 81.6% |

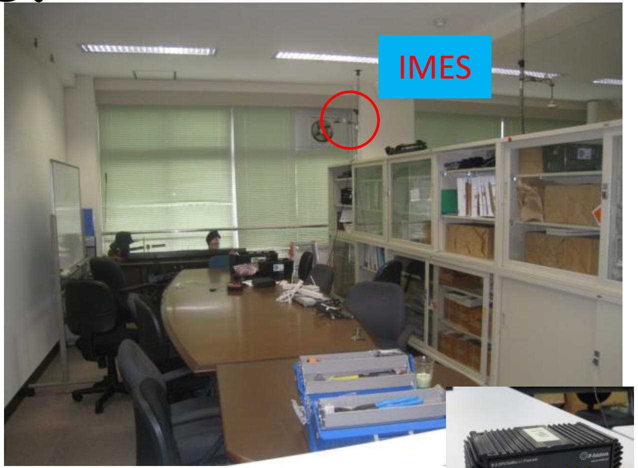
Shinjuku

Route 20

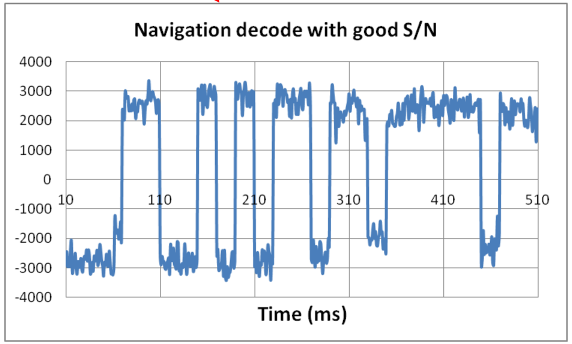
TUMSAT

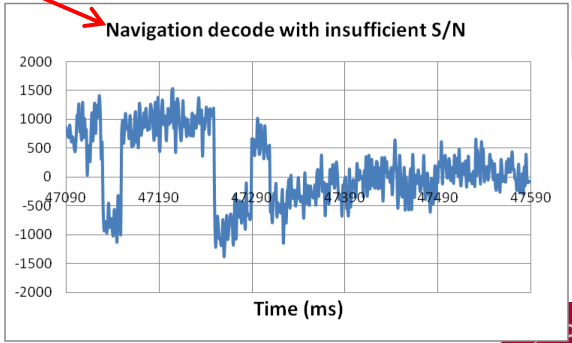
# How about indoor?
# 5 min IMES tracking in Lab.




Front-end

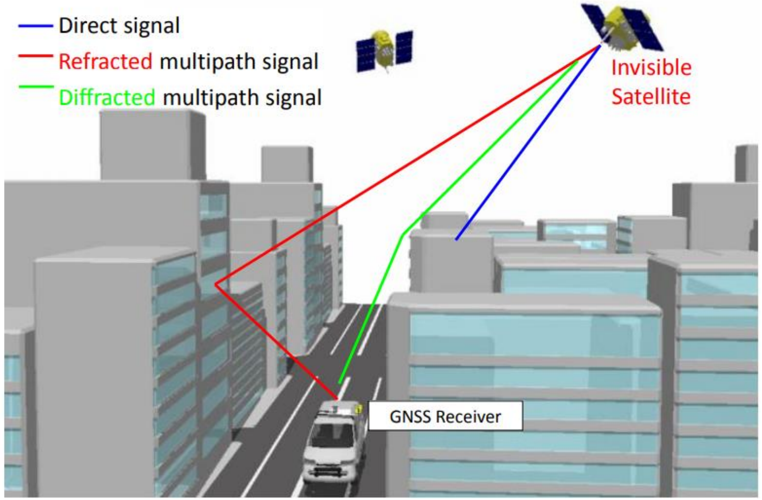


# Improve GNSS Positioning

> **3D Mapping Aided (3DMA) GNSS:**

- Suzuki, Taro and Kubo, Nobuaki. GNSS positioning with multipath simulation using 3D surface model in urban canyon. (*ION GNSS+ 2012*). (GNSS NLOS exclusion causes poor satellite geometry)

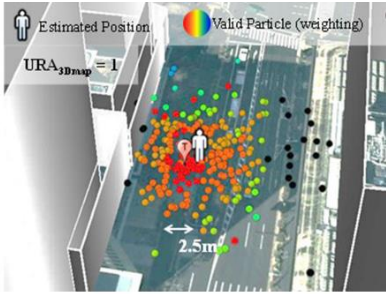



Prof. Kubo, 2012

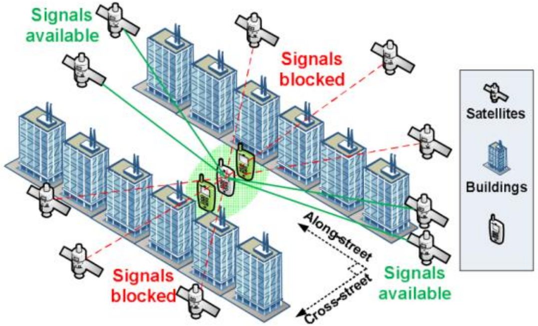
# Improve GNSS Positioning

> **3D Mapping Aided (3DMA) GNSS:**

- Wang, Lei, et al. "Urban positioning on a smartphone: Real-time shadow matching using GNSS and 3D city models." *Navigation, The Institute of Navigation*, 2013.(Relies on the satellite visibility classification and the initial guess of the receiver)
- Hsu, Li-Ta, et al. "3D building model-based pedestrian positioning method using GPS/GLONASS/QZSS and its reliability calculation." *GPS solutions*, 2016.(Relies on the initial guess of the receiver and causes high computation load)




Hsu, 2016



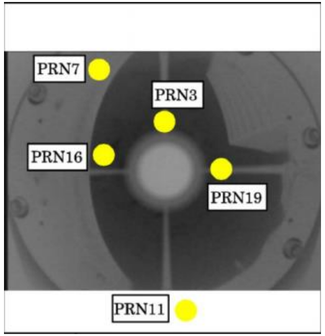

Wang, 2013

# Improve GNSS Positioning

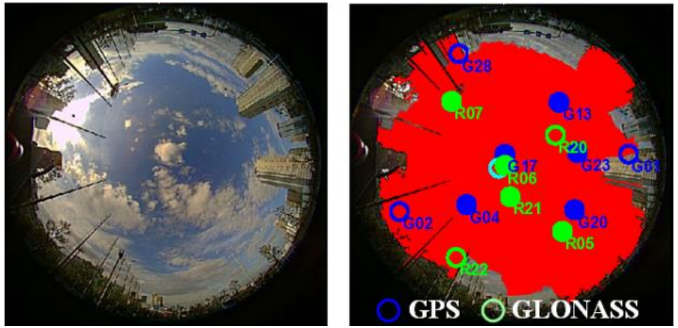
> **Camera-aided GNSS Positioning:**

- Meguro, Jun-ichi, et al. "GPS multipath mitigation for urban area using omnidirectional infrared camera." *IEEE Transactions on Intelligent Transportation Systems*, 2013. (NLOS exclusion cause poor geometry)

- Suzuki, Taro and Kubo, Nobuaki, "N-LOS GNSS Signal Detection Using Fish-Eye Camera for Vehicle Navigation in Urban Environments," (*ION GNSS*+ 2014), *Tampa*, *Florida*, September 2014. (NLOS detection and exclusion with monocular camera, cause poor satellite geometry)



Meguro, 2013



Kubo, 2014

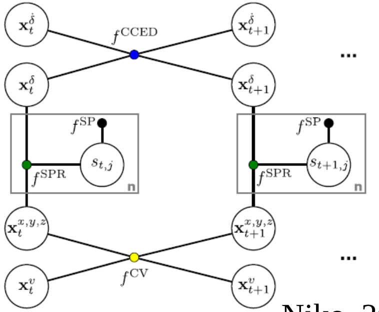
# Improve GNSS Positioning

## > **Robust Model-aided GNSS Positioning:**

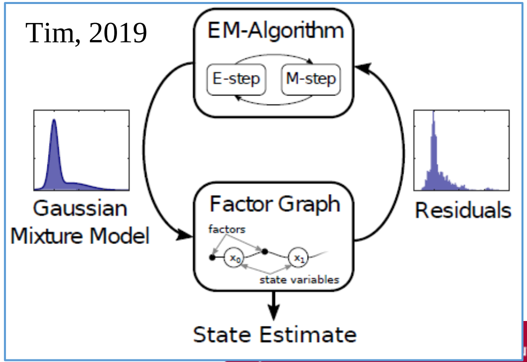
- Sünderhauf, et al. "Switchable constraints and incremental smoothing for online mitigation of non-line-of-sight and multipath effects. *IEEE IV* 2013. (Relies on the initial guess of the prior factor)

- Pfeifer, Tim, et al. "Dynamic Covariance Estimation—A parameter free approach to robust Sensor Fusion." *IEEE MFI* 2017. (Relies on the percentage of healthy measurements)

- Pfeifer, Tim, and Peter Protzel. "Expectation-maximization for adaptive mixture models in graph optimization." *ICRA*, 2019. (Relies on the initial guess of the state estimation)




Niko, 2013



# References


- Chapters 5 and 6.1-4 - Collinson R.P.G., *Introduction to Avionics Systems*, *Third Edition*, Springer, Feb 2011

- Chapter 2, Paul D. Groves, *Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems, 2nd Edition*, Artech House, 2013.

# Q&A

## Thank you for your attention ☺

## Q&A

Dr. Weisong Wen

If you have any questions or inquiries, please feel free to contact me.

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