Department of Aeronautical and Aviation Engineering
航空及民航工程學系

THE HONG KONG POLYTECHNIC UNIVERSITY
香港理工大學

Fundamental Concepts for Guidance and Navigation

AAE4203 – Guidance and Navigation

Dr Weisong Wen
Research Assistant Professor

Department of Aeronautical and Aviation Engineering
The Hong Kong Polytechnic University

Week 1, 12 Jan 2022

Dr WEN Weisong

THE HONG KONG POLYTECHNIC UNIVERSITY 香港理工大學

2021~Now, Research Assistant Professor in Navigation, Autonomous Driving

Berkeley UNIVERSITY OF CALIFORNIA

2018, Visiting Researcher in Navigation, Autonomous Driving

THE HONG KONG POLYTECHNIC UNIVERSITY 香港理工大學

2020, PhD, Senior Research Fellow (2021 in Navigation, Positioning, Robotics

智行者 IDRIVERPLUS

2016, Research algorithm in Navigation, Autonomous Driving (Industry experience)

ION INSTITUTE OF NAVIGATION

IEEE ITSS

IEEE Robotics & Automation Society

- 19+ SCI Journal paper publications in Navigation.
- 23+ Conference papers in Navigation.
- Session chairs of ION GNSS+ in Navigation.
- Session chairs of PolyU-TUM Workshop.
- Regular reviewer in IEEE T-ITS (2017~), IEEE ITSM (2017~), IEEE Sensors Journal (2018~), IEEE T-VT (2017~) in Navigation.

Developed ***navigation*** technology was applied to Huawei Technologies' products

How About You?

- Let’s get to know with each other
    - Short introduction about yourself? ☺
    - Who is your *Final Year Project* supervisor and what is your topic? ☺
    - Why you select this course? ☺

Ground Rules

For students:

Open mind; speak English; participate activities assigned; ask questions

For teachers:

Arrive on time; reply emails on time; answer questions related to the subject

Be curious, Be inspired, Be motivated, Study further by yourself.

Intended Learning Outcomes

- Understand and explain the working principles of navigation and guidance systems for air vehicles; and
- Competently apply the fundamental mathematical concepts of aircraft navigation; and
- Critically evaluate the characteristics, purposes, and design procedures of aircraft navigation and guidance systems; and
- Identify the technological and design trends of future aircraft navigation.

Assessment

- Homework Assignment (15%)
- Mid-Term Quiz (15%)
- Group Project (Case study) (30%)
- Final Exam (50%)

Teaching Plan

Teaching Plan of AAE4203 (2021/2022 Semester 2)

1. Subject Title and Subject Code

Guidance and Navigation (AAE4203)

2. Enrolment and Class Size

3. Subject Intended Learning Outcome (ILO)

a. To provide a fundamental understanding and knowledge of conventional and modern design and working principles of navigation and guidance for air vehicles.
b. To provide the basic mathematical concepts of navigation by sensors, including inertial, visual, LiDAR, and satellite approaches and guidance laws.
c. To provide an expansive view into the technological trends of future aircraft navigation and guidance systems designs.

4. Grading Policy:

- **Mid-Term Quiz** (15%)
- **Homework Assignment** (15%)
- **Group Project** (20%)
- **Examination** (50%)

TENTATIVE TEACHING PLAN

AAE4203 – Guidance and Navigation

Plan of Teaching, Learning, and Assessment – 1st Semester 2020/2021

Instructors: Dr. Weisong WEN, PQ408, welson.wen@polyu.edu.hk

Teaching Assistant: Mr. LIU Ruirui, Miss. LEE Pin Hsun

Venue and time: Lecture: 15:30-18:20 (Wed) (Face-to-Face), R602

Sem. Week	Topics Taught	Planned Learning Outcomes	Assessment	Timetable, Venue Staff
Wk1 (Jan 12)	Fundamental concepts	The concept of navigation and positioning, coordinate frame; the different methods of representing attitude, rotation, and resolving axes transformations		Wed 15:30-18:20 (Face-to-Face) R602 Dr. Weisong WEN
Wk 2 (Jan 19)	Inertial Navigation: From Sensor to Modeling	The basic principles of accelerometer, gyro, and IMU technology; the main concepts of inertial navigation		Wed 15:30-18:20 (Face-to-Face) R602 Dr. Weisong WEN
Wk 3 (Jan 26)	Satellite Navigation	Principles of satellite navigation; ICAO requirements on accuracy, integrity, continuity, and availability; stand-alone positioning, nominal and erroneous errors		Wed 15:30-18:20 (Face-to-Face) R602 Dr. Weisong WEN
Wk 4 (Feb 9)	State Estimation: Kalman Filtering for Integrated Navigation	The basic principle of state estimation. State, measurement model, and Kalman filtering		Wed 15:30-18:20 (Face-to-Face) R602 Dr. Weisong WEN
Wk5 (Feb 16)	Visual Navigation: From Sensor to Modeling I	The basic principles of the camera, image model, and feature descriptors		Wed 15:30-18:20 (Face-to-Face) R602 Dr. Weisong WEN
Wk6 (Feb 23)	Tutorial on visual feature detection and tracking	Demonstration of the visual feature detection, tracking and matching using real image data		Wed 15:30-18:20 (Face-to-Face) QT004 Dr. Weisong WEN
Wk7 (Mar 2)	Mid-term	Quiz on week 1-6 subjects contents		Wed 15:30-18:20 (Face-to-Face) R602 Dr. Weisong WEN
Wk8 (Mar 9)	Visual Navigation: From Sensor to Modeling II	The basic principles of visual matching, feature tracking, and visual positioning		Wed 15:30-18:20 (Face-to-Face) R602 Dr. Weisong WEN
Wk 9 (Mar 16)	Tutorial on visual positioning	Demonstration of the visual odometry based positioning using real image data		Wed 15:30-18:20 (Face-to-Face) QT004 Dr. Weisong WEN
Wk 10 (Mar 23)	State Estimation: Factor Graph for Integrated Navigation I	The basic principle of the factor graph, modeling, least-square estimation.		Wed 15:30-18:20 (Face-to-Face) R602 Dr. Weisong WEN
Wk 11 (Mar 30)	State Estimation: Factor Graph for Integrated Navigation II	The basic principle of optimization with the Gauss-Newton algorithm. Application of the factor graph in GNSS positioning.		Wed 15:30-18:20 (Face-to-Face) R602 Dr. Weisong WEN
Wk 12 (Apr 6)	Case Study Presentation	Self-learning on various topics on guidance and navigation	Group Presentation & Report	Wed 15:30-18:20 (Face-to-Face) R602 Dr. Weisong WEN
Wk 13 (Apr 13)	Case Study Presentation	Self-learning on various topics on guidance and navigation	Group Presentation & Report	Wed 15:30-18:20 (Face-to-Face) R602 Dr. Weisong WEN

Updated on 2019/09/01

Updated on 2019/09/01

See the full version of the teaching plan.

Outline

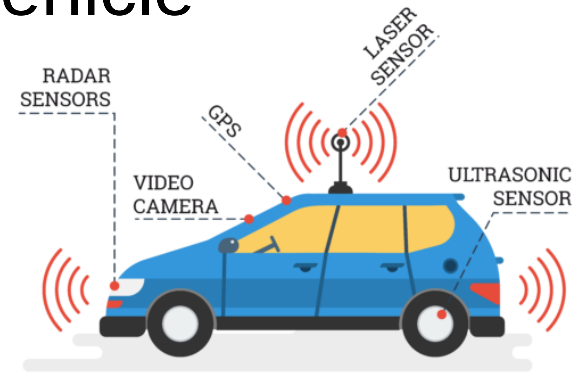
- Introduce the concept of typical navigation methods and applications.
- Describe the main coordinate frames used in navigation
- Explains the representation of the position, rotation, and resolving axes transformations, and shows how to convert between them.

Navigation Methods and Applications

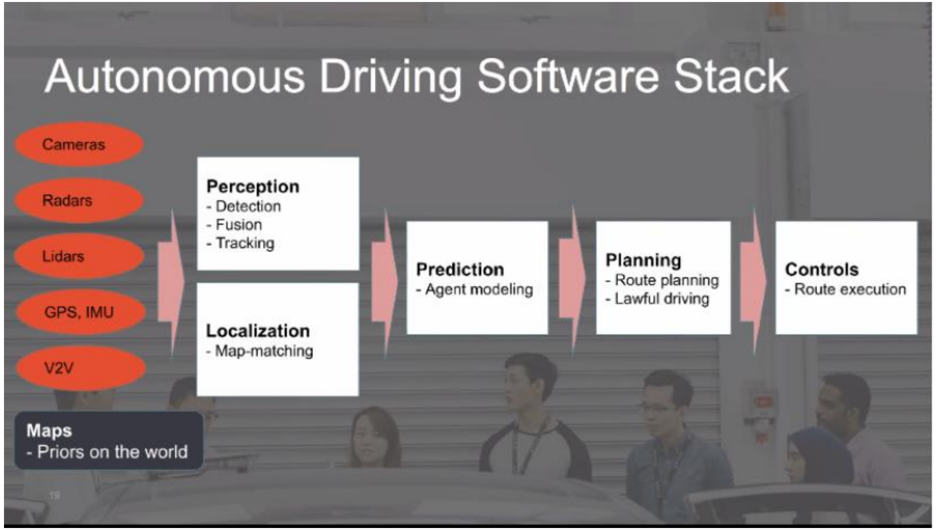
Navigation in Autonomous Driving Vehicle

Tesla Autonomous Driving Car

https://www.youtube.com/watch?v=tlThdr3O5Qo



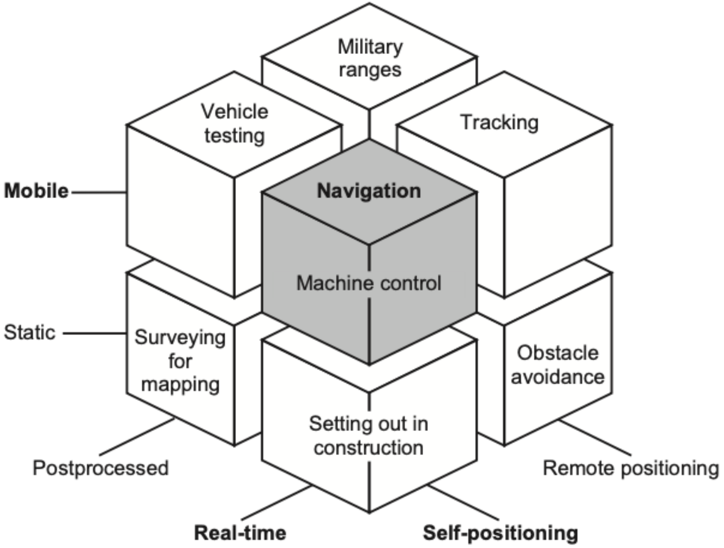
Integration of cameras, maps, vehicle sensors and GNSS for robust and accurate navigation.



Navigation

Definition: "any of several methods of determining or planning a ship's or aircraft's position and course by geometry, astronomy, radio signals, etc."

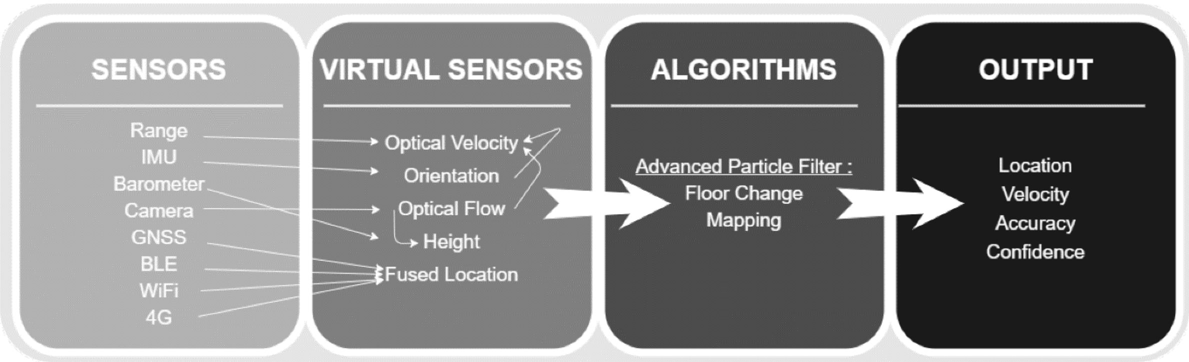
List a few examples of navigation devices that you are using in every life.

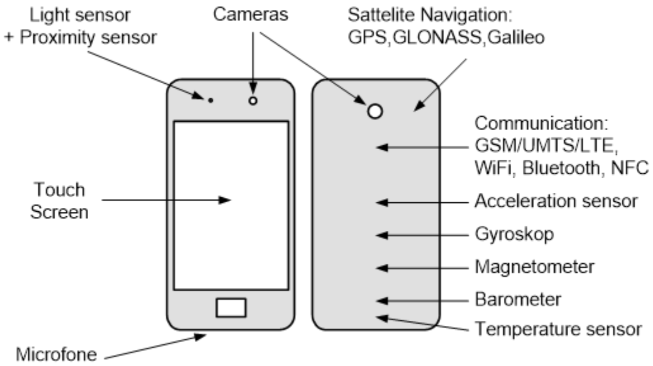


Navigation

Definition: “any of several methods of determining or planning a ship’s or aircraft’s position and course by geometry, astronomy, radio signals, etc”

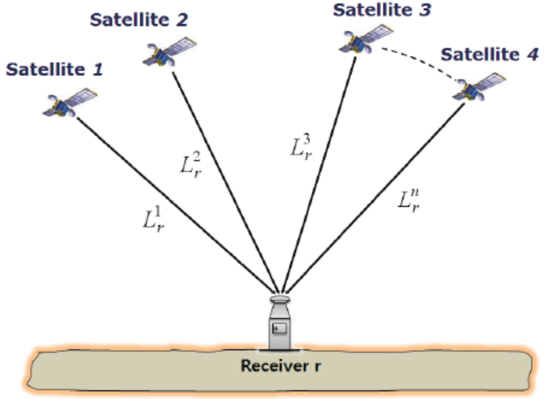
What sensors do we have in a smartphone for navigation?

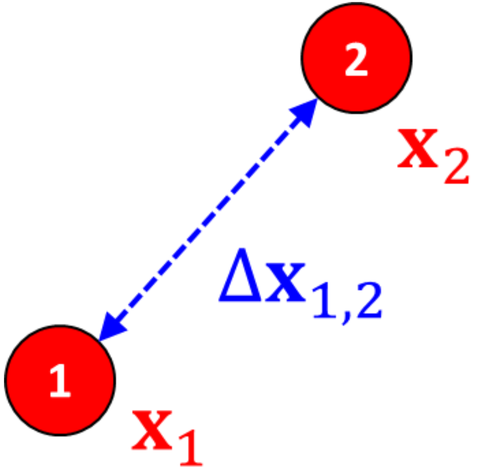




Sensors for Navigation

- Radio navigation (**point-positioning**)
  - Ultra wideband (UWB)
  - Wi-Fi
  - Cell communication 3G/4G/5G
  - Satellite navigation
  - etc
- Robotics navigation* **(dead-reckoning)**
  - Inertial sensors, accelerometers/gyroscope/magnetometer
  - Visual sensors
  - LiDAR sensors
  - etc



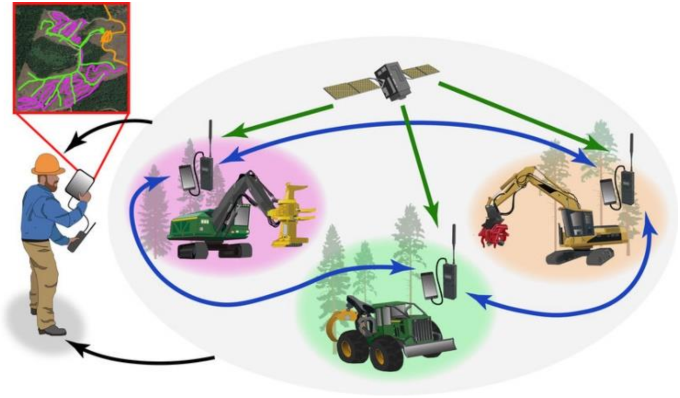


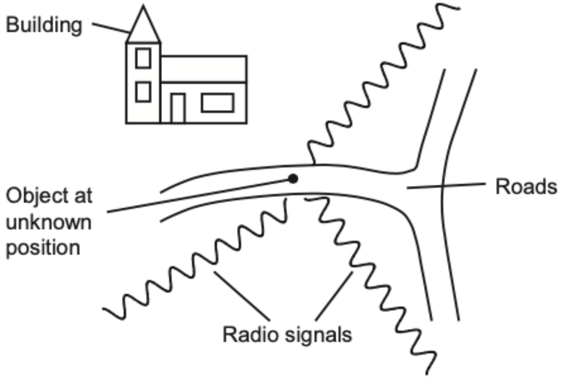
Framework of Sensors to Navigation

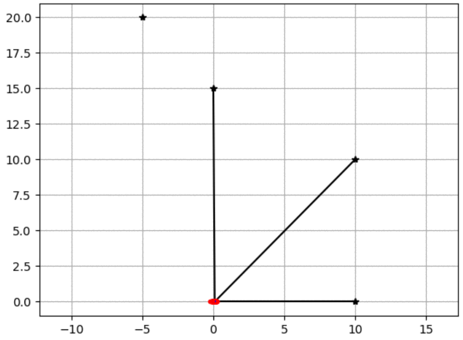
	Sensing	Measuring	Measurement	Model	Estimate
GNSS	GNSS Signal	Signal Defined Receiver	Pseudorange	Least Square Estimation	Point Position
INS	Raw measurements (voltage, delays, etc)	Accelerometer/ Gyroscope Models	Acceleration & Angular Velocity	INS Mechanization	Relative Position
Visual Navigation	Images	Features Detection & Tracking	Associated Features	Least Square Estimation	Relative Position

Point Positioning Methods

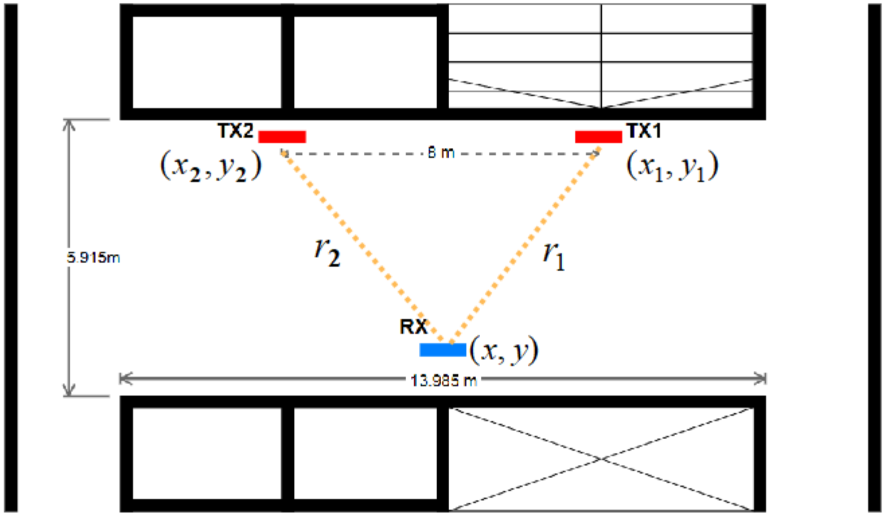
- Position may be inferred directly by matching the signals receivable and/or features observable at a given location with a database.
- Alternatively, more distant landmarks at known positions may be selected and their distance and/or direction from the user measured. A landmark may be a transmitter (or receiver) of signals or an environmental feature. A landmark installed specifically for navigation is known as an aid to navigation







Example: TDOA (Time Difference of Arrival) Model



$$(x_1 - x)^2 + (y_1 - y)^2 = r_1^2$$
$$(x_2 - x)^2 + (y_2 - y)^2 = r_2^2$$

$$x_i^2 + y_i^2 + x^2 + y^2 - 2x_i x - 2y_i y = r_i^2$$

$$K_i = x_i^2 + y_i^2, \quad R = x^2 + y^2$$

$$r_i^2 - K_i = -2x_i x - 2y_i y + R$$

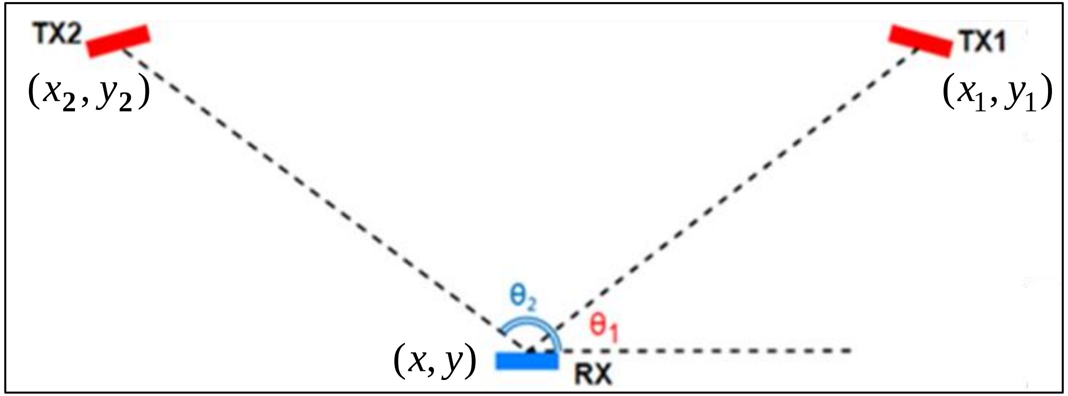
$$\begin{bmatrix} r_1^2 - K_1 \\ r_2^2 - K_2 \end{bmatrix} = \begin{bmatrix} -2x_1 & -2y_1 & 1 \\ -2x_2 & -2y_2 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ R \end{bmatrix}$$

$$\mathbf{Ax} = \mathbf{b}$$

1. The initial guess is given by Linear Least Square.
2. Solving the Nonlinear Least Square by Levenberg-Marquardt Method
3. Different selection strategies:
   (a) 2000 data one-one correspondence
   (b)Minimun Delay in 10 channel (200 data)
   (c)Delay with the maximum Delay in 10 channel (200 data)

How many signals are required to calculate the position of the receiver?

Example: AOA (Angle of Arrival) Model



$$\frac{y_i - y}{x_i - x} = \tan\theta_i$$

$$\begin{cases}(y_1 - y) = (x_1 - x)\tan\theta_1 \\ (y_2 - y) = (x_2 - x)\tan\theta_2\end{cases}$$

$$\begin{bmatrix}\tan\theta_1 & 1 \\ \tan\theta_2 & 1\end{bmatrix}\begin{bmatrix}x \\ y\end{bmatrix} = \begin{bmatrix}x_1\tan\theta_1 + y_1 \\ x_2\tan\theta_2 + y_2\end{bmatrix}$$

$$\mathbf{Ax} = \mathbf{b}$$

1. We choose data of Point A for analysis.
   The initial guess is given by Linear Least Square.

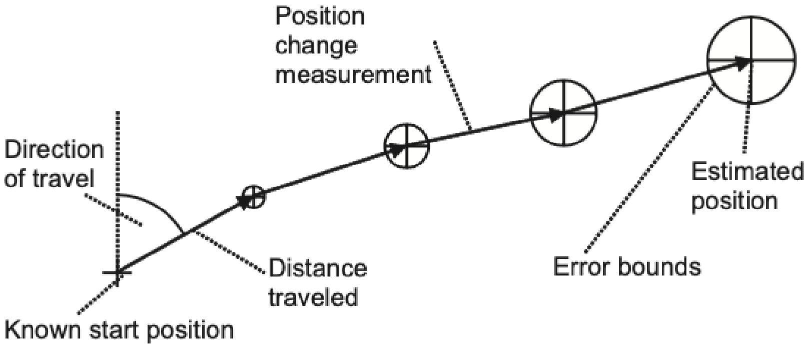
2. Solving the Nonlinear Least Square by Levenberg-Marquardt Method

3. Different selection strategies: (a) 2000 data one-one correspondence
   (b)Filter the data larger than 180° or less than 0° situation
   (c)Angle with the minimun Delay in 10 channel (200 data)
   (d)Angle with the maximun Gain in 10 channel (200 data)

Dead Reckoning

- Dead reckoning either measures the <u>change in position or measures the velocity and integrates it.</u> Therefore, if the initial position is known, the current position may be determined as shown in the figure.
- For two-dimensional navigation, a heading measurement is sufficient, whereas for three-dimensional navigation, a full three-component attitude measurement is needed.

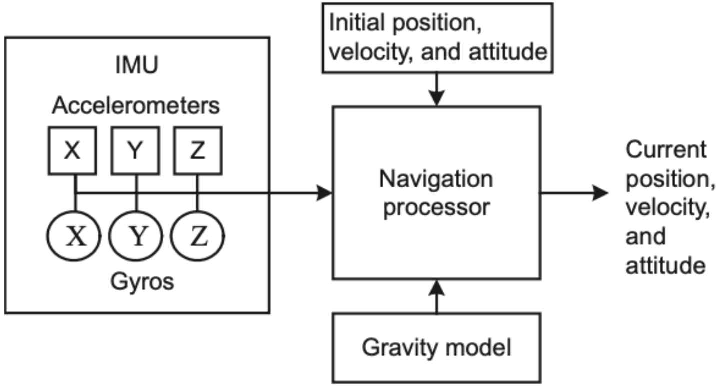
Heading may be measured using a *<u>magnetic compass</u>*. This is an ancient technology, although today magnetic compasses and magnetometers are available with electronic readouts.



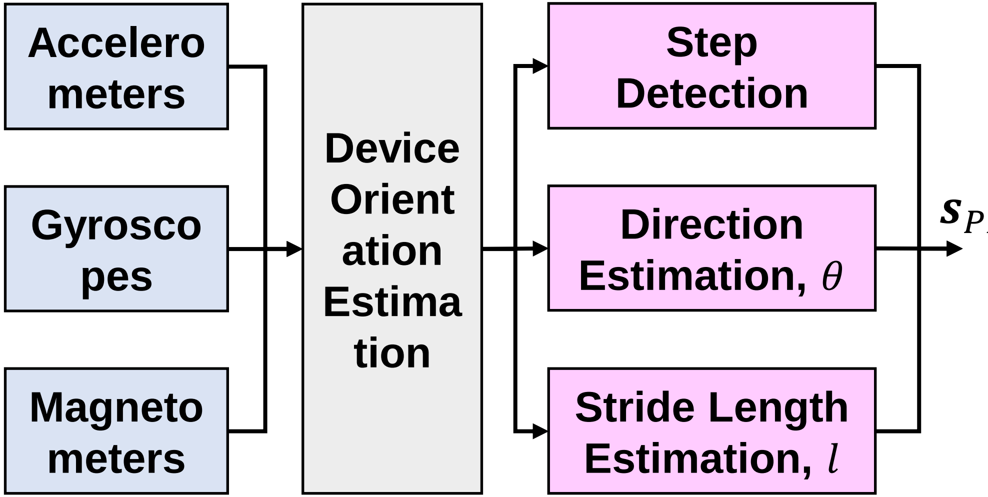
Dead Reckoning

- For three-dimensional navigation applications, the roll and pitch components of attitude may be determined by using *accelerometers* or a tilt sensor to determine the direction of gravity or from a horizon sensor.
- Finally, *gyroscopes* (gyros), which measure angular rate, may be used to measure changes in attitude.

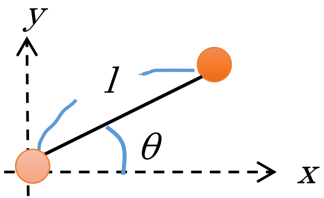
An inertial navigation system (INS) is a complete three-dimensional dead-reckoning navigation system. It comprises a set of inertial sensors, known as an *inertial measurement unit* (IMU), together with a navigation processor.

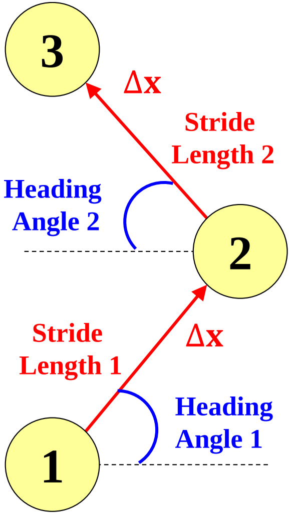


Pedestrian Dead Reckoning (PDR)



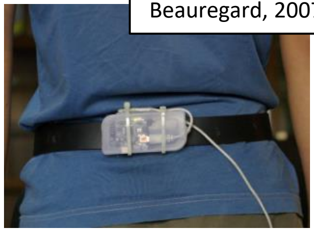
Positioning Update





$$\Delta\mathbf{x} = \begin{bmatrix} x_{east} \\ x_{north} \end{bmatrix} = f_{PDR}(\boldsymbol{s}_{PDR}) = step_{dct} \cdot \begin{bmatrix} l \cdot cos\theta \\ l \cdot sin\theta \end{bmatrix}$$

$$\boldsymbol{s}_{PDR} = \begin{bmatrix} l \\ \theta \end{bmatrix}, step_{dct} \in \langle 0,1 \rangle \qquad \mathbf{x}_n = \mathbf{x}_{n-1} + \Delta\mathbf{x}$$


Beauregard, 2007 [6]

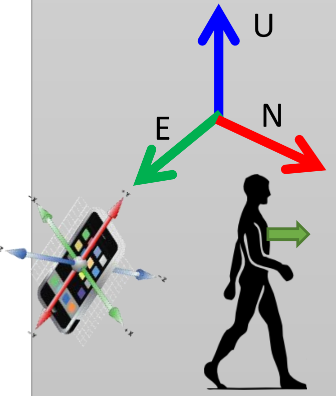

Goyal et al.,2011 [5]


Kamisaka et al., 2011 [8]


Steinhoff et al., 2010 [7]

Pedestrian Dead Reckoning (PDR)

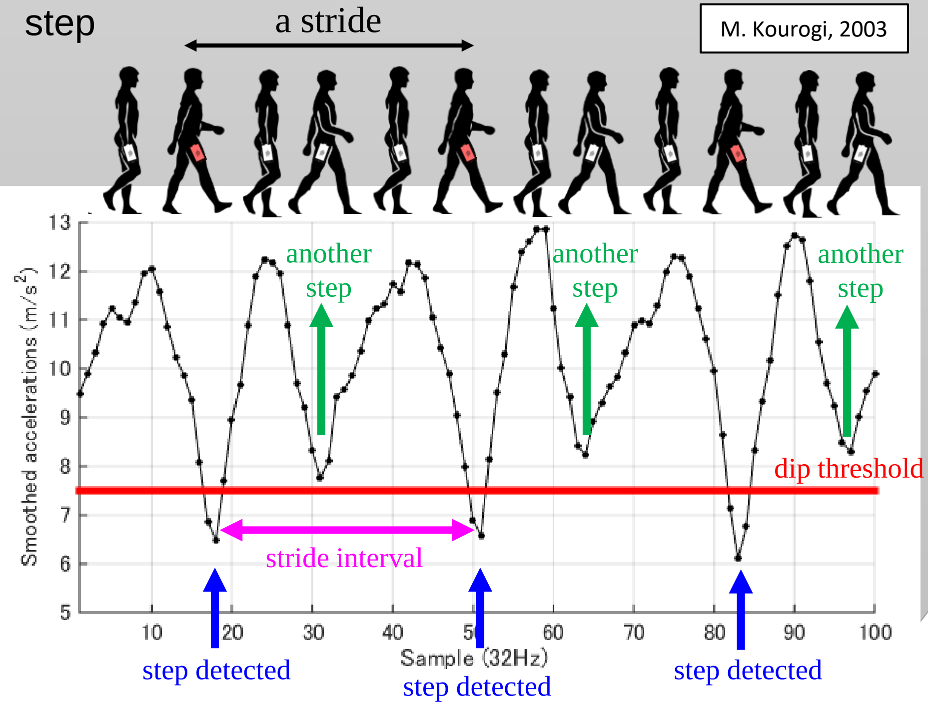
Device Orientation Estimation



Transform The X-Y-Z (ECEF) measured data to E-N-U (Local) Coordinate

Step Detection

Detect the dip of the vertical acceleration data as one step

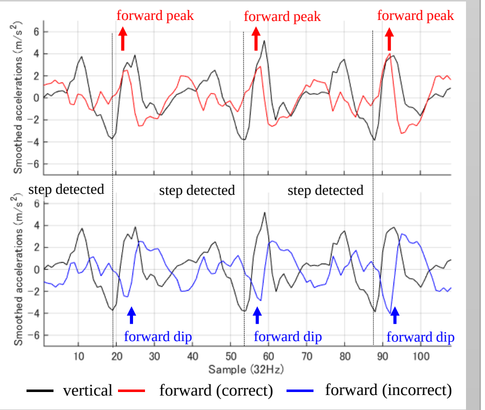


Stride Length Estimation

Using vertical acceleration (Weinberg equation [2])

$$l = K\sqrt[4]{a_{v,\max} - a_{v,\min}}$$

Moving Direction Estimation



Applying PCA on acceleration in E-N plane (Steinhoff et al.'s method [1])

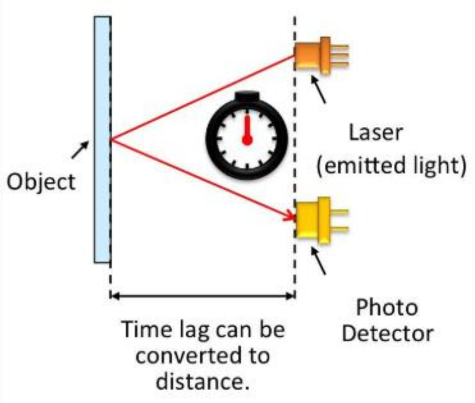
[1] H. Weinberg "Using the ADXL202 in Pedometer and Personal Navigation Applications," Analog Devices Inc. Application Note, 2002
[2] U. Steinhoff and B. Schiele "Dead Reckoning from the Pocket - An Experimental Study," *PerCom*, 2010

Light Detection and Ranging (LiDAR)

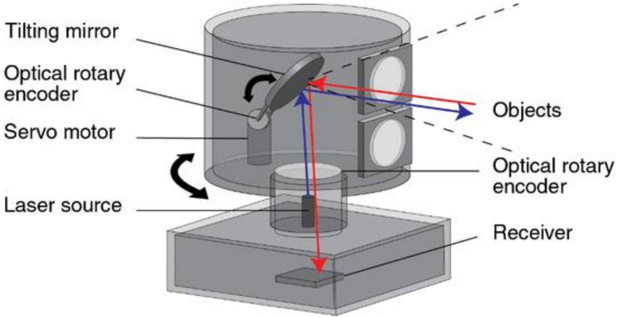
Laser signal emitted from a LiDAR reflect from objects both on and above the ground surface: vegetation, buildings, bridges, and so on. One emitted laser pulse can return to the LiDAR sensor as one or many returns.

The distance measurement's equation is given below,

Distance = (Speed of Light x Time of Flight) / 2



Multiple Channel LiDAR



Source: Single Channel LiDAR

Source: Multiple Channel Rotating LiDAR

Light Detection and Ranging (LiDAR)

2007

The HDL-64E becomes the first commercially available
0.6 Million HK Dollar

Google (now Alphabet) begins testing self-driving cars on San Francisco Bay Area streets using Velodyne's lidar technology. Alphabet's first self-driving car prototype uses Velodyne's HDL-64E lidar sensor

2014

Velodyne launched the Pucks series

QUANERGY

OUSTER

HESAI

2016

Velodyne expands the Puck family with the launches of three sensors: Puck Lite, Puck Hi-Res, and Ultra Puck.

History of The Mainstream 3D LiDAR

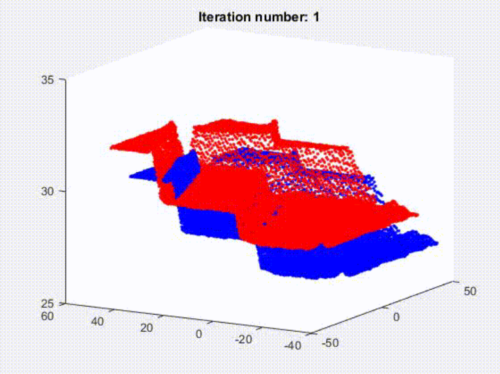
Solid-state LiDAR attracted increasing attention
10K HK Dollar

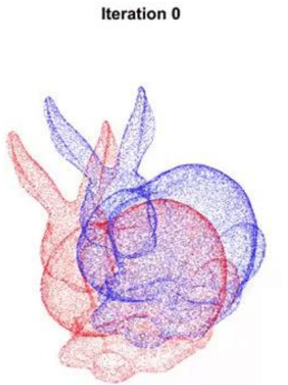
Prices of the Mainstream LiDAR

	Product	*Inc*	*Type*	*Price*
	HDL-64E	Velodyne	Mechanical LiDAR	**$75,000**
	HDL-32E	Velodyne	Mechanical LiDAR	**$30,000**
	VLP-16	Velodyne	Mechanical LiDAR	**$4,000**
	OS1	Ouster	Mechanical LiDAR	**$3,500**
	Horizon	Livox	Solid-State LiDAR	**$800**
	Velarray H800	Velodyne	Solid-State LiDAR	**$500**

LiDAR for autonomous driving is getting cheaper and cheaper!

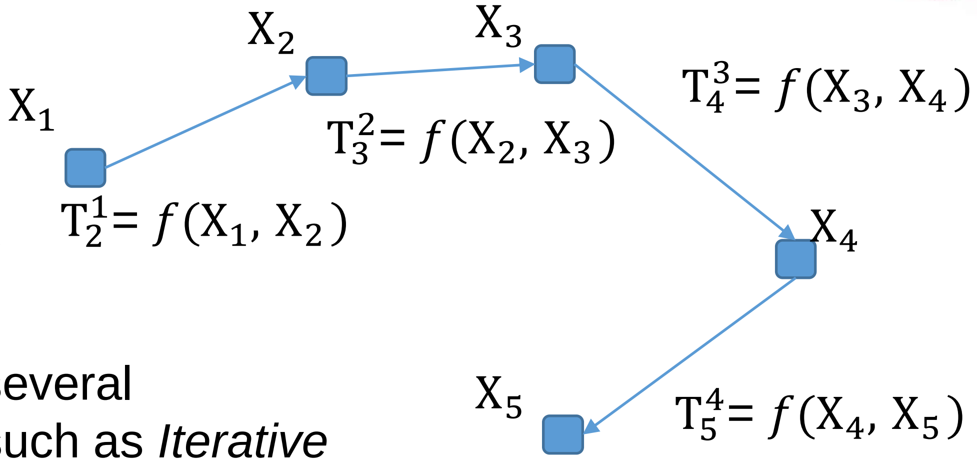
LiDAR Localization Method





X_{k+1} and X_k are two consecutive frames of point cloud

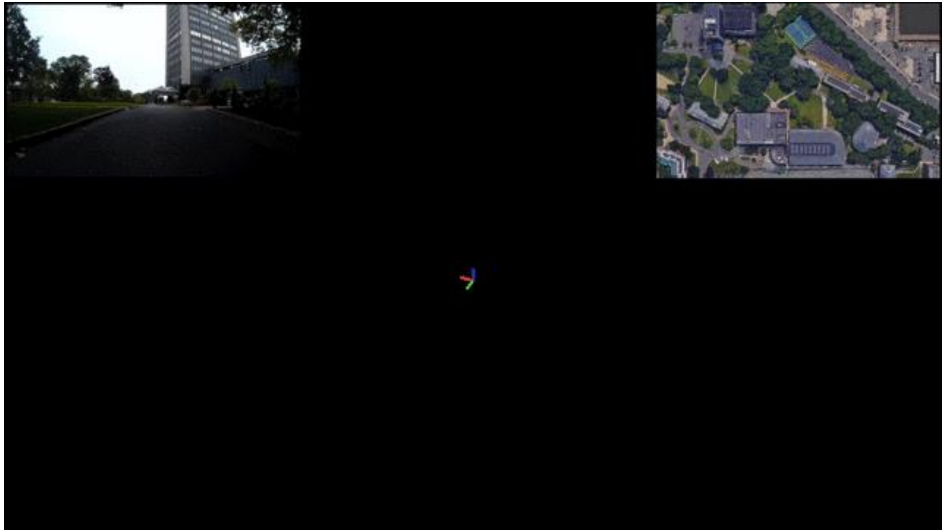
$$T_{k+1}^{k} = f(X_k, X_{k+1})$$

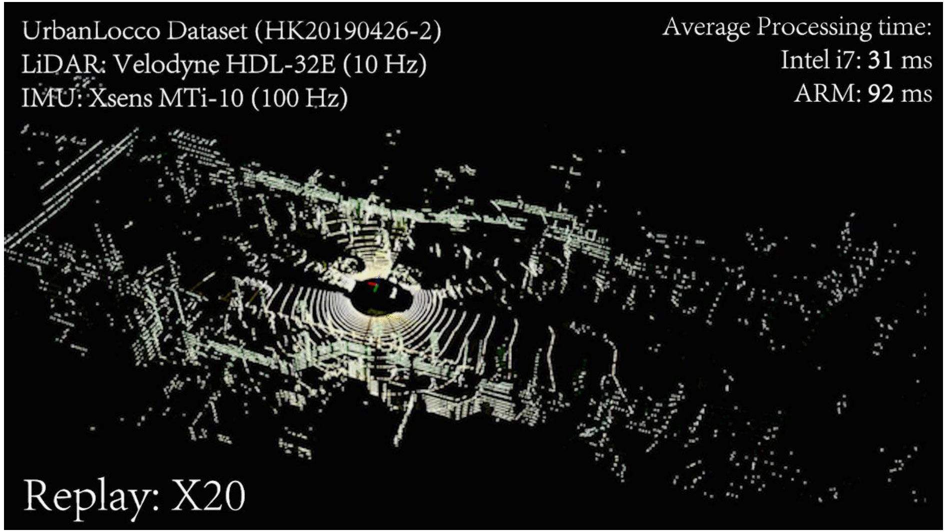


The function $f(X_k, X_{k+1})$ denotes several algorithms for LiDAR localization, such as *Iterative Closest Point*(***ICP***), *Normal Distributions Transform*(***NDT***),et al.

Huang, F., Wen, W., Zhang, J., & Hsu, L. T. (2021). Point wise or Feature wise? Benchmark Comparison of Public Available LiDAR Odometry Algorithms in Urban Canyons. *arXiv preprint arXiv:2104.05203*.

LiDAR Localization Method

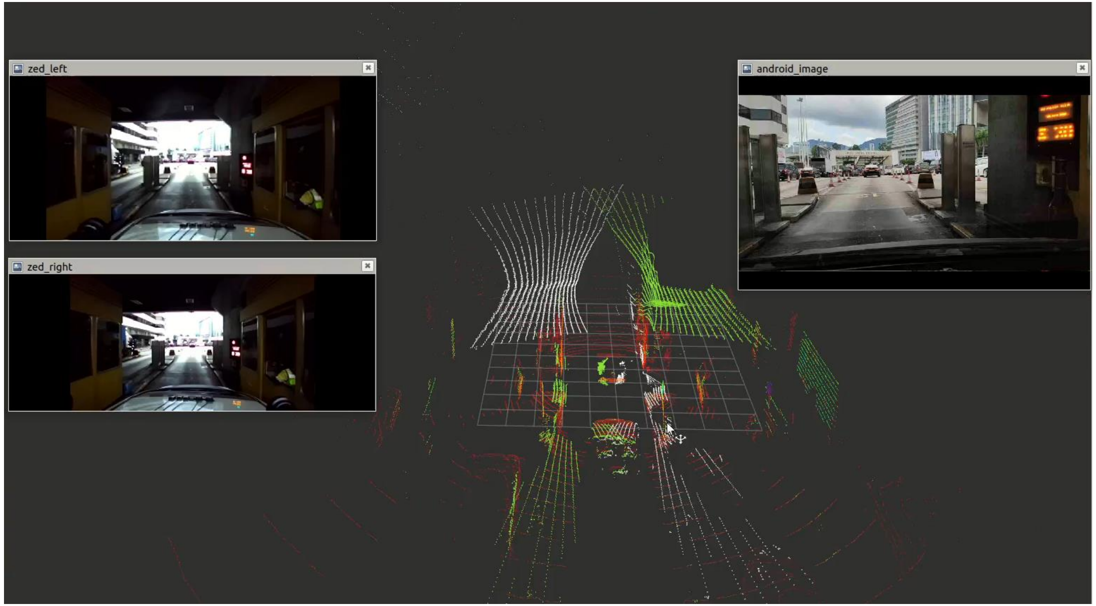




Pros of LiDAR localization: very accurate and high frequency pose estimation over time. The map of the environment can be generated simultaneously.

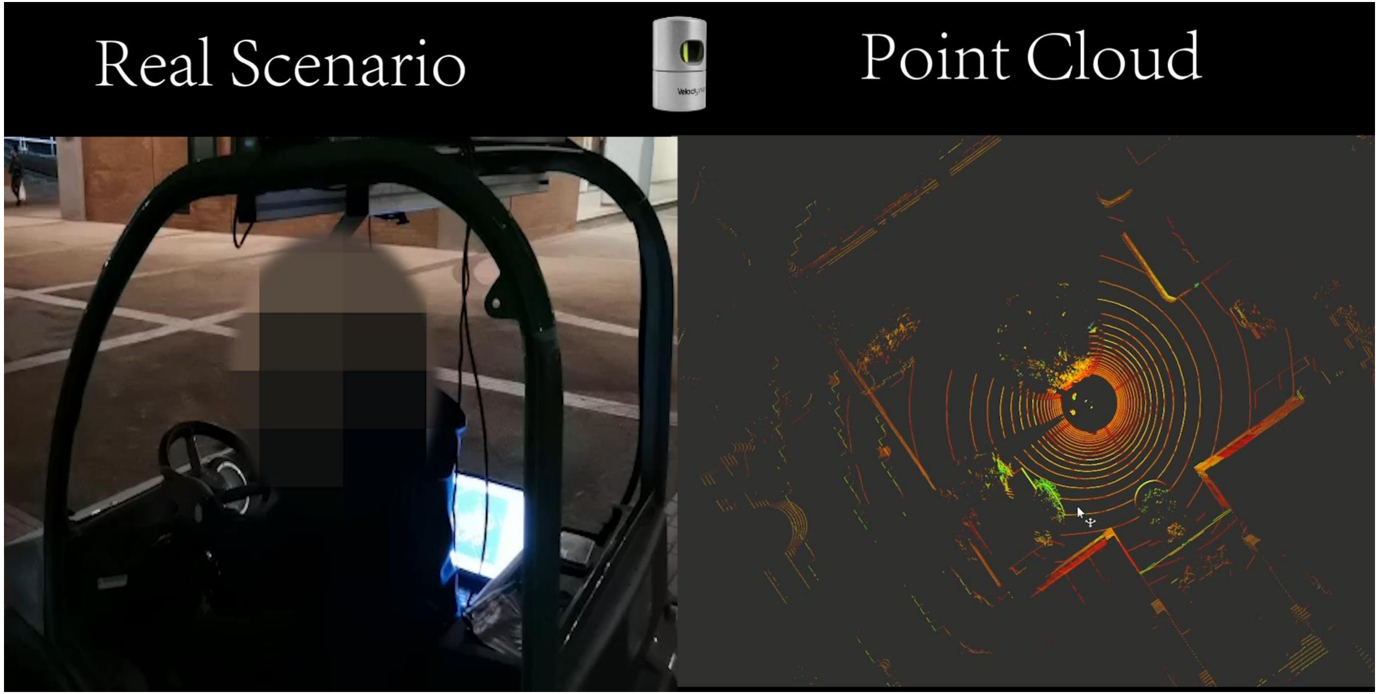
Cons of LiDAR localization: The localization result is subject to drift over time. The LiDAR localization can fail in feature insufficient environments.

Multiple LiDARs for Autonomous Vehicle Navigation



Testing of the multiple 3D LiDARs for typical autonomous vehicles setups.

LiDAR for Autonomous Vehicle Navigation



Testing of the autonomous driving vehicles in PolyU campus.
Only a 3D LiDAR is employed for navigation.

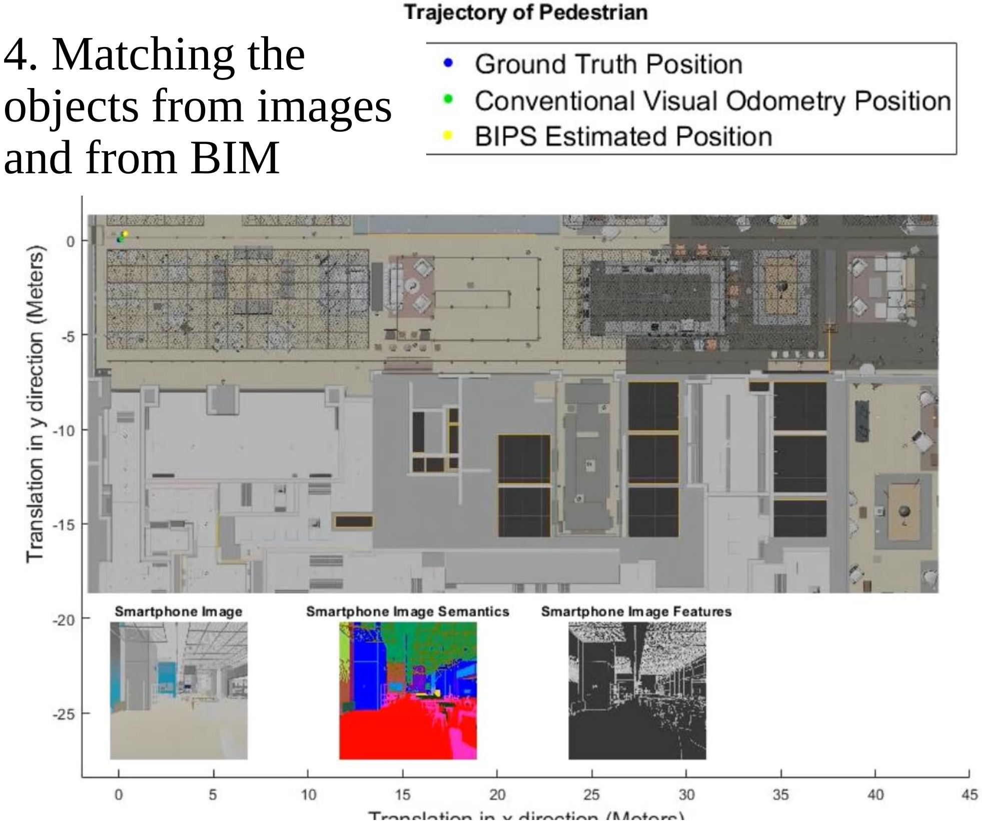
Visual indoor positioning – semantic information

1. Taking images to get ready for object recognition

2 Object recognition and classification based on deep learning

3. Object extraction from building information model (BIM) from grids

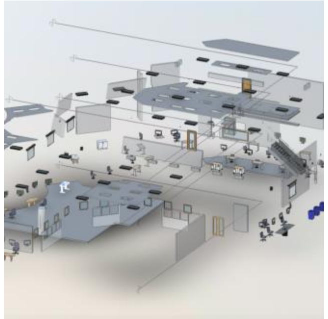
4. Matching the objects from images and from BIM

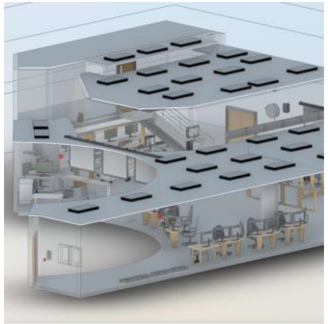


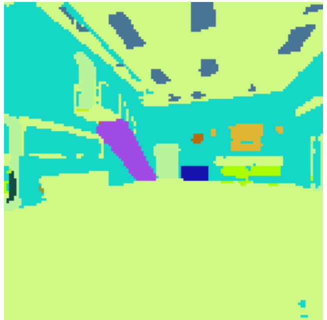
M. Lee, H.-Y. Ho, **Li-Ta Hsu***, S. Au, BIPS: Building Information Positioning System, *IEEE IPIN 2021* (accepted)

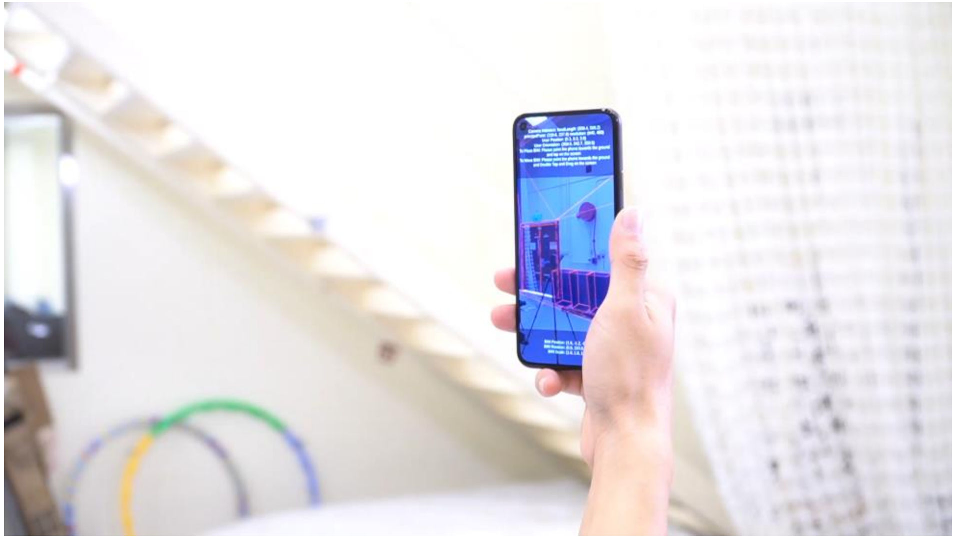
Visual Indoor Positioning with Augmented Reality (AR)

Server setup with BIM → Turn on Camera → Point at Asset → Instant asset position and informatics!



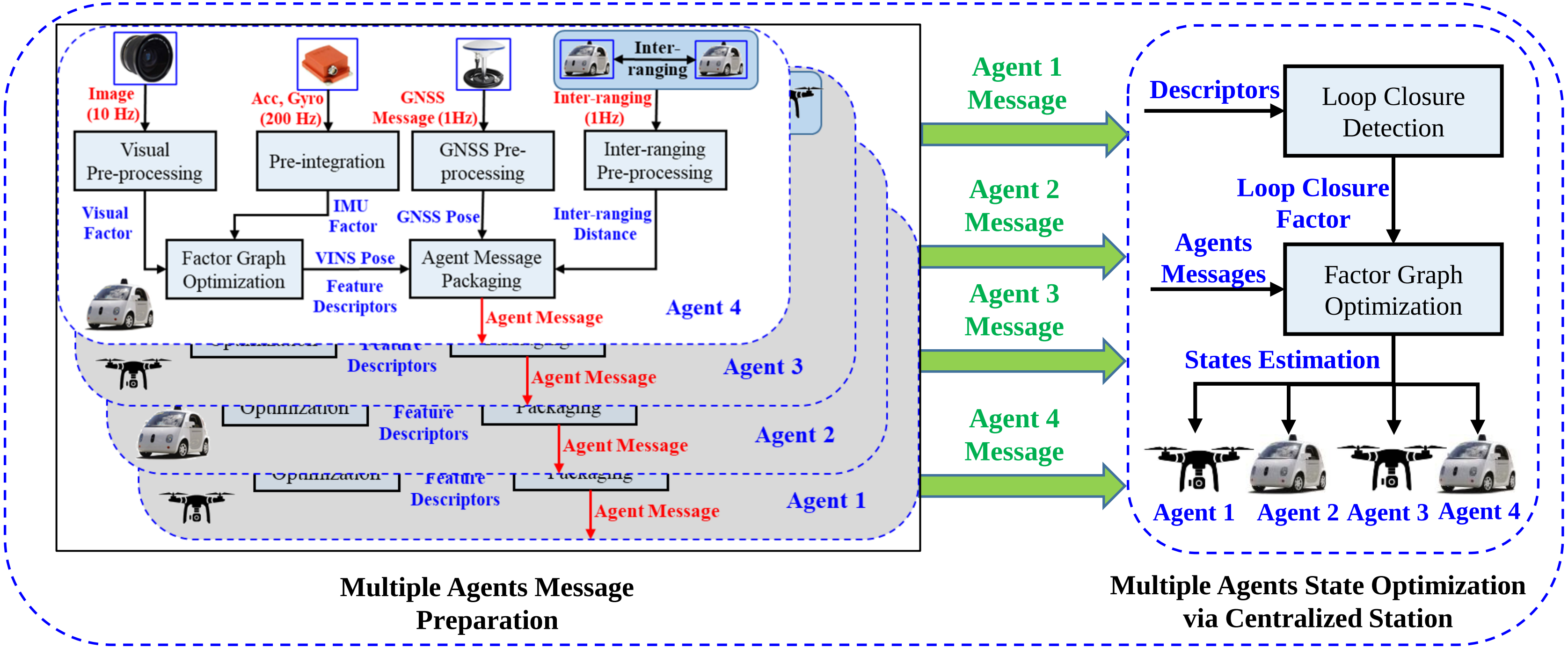




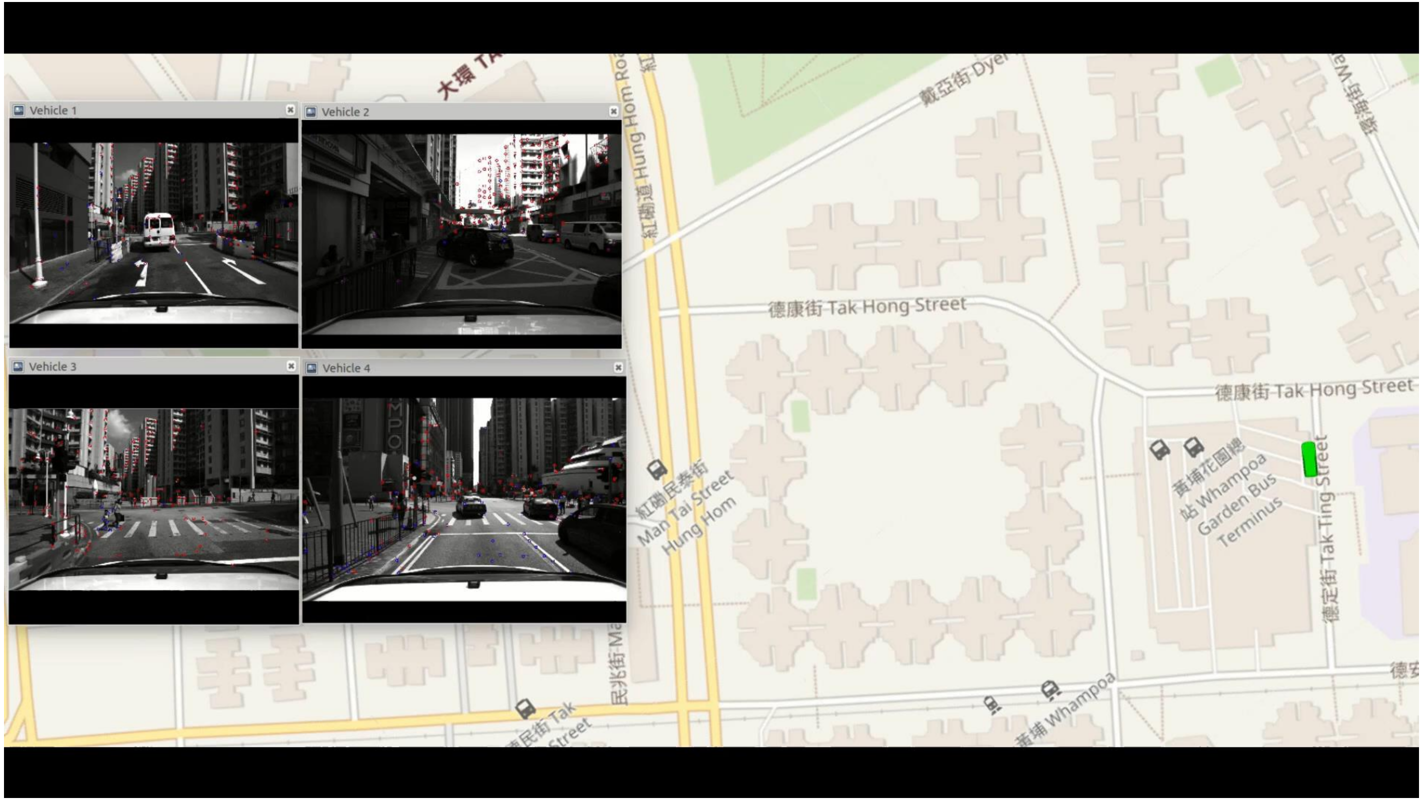


Multi-agents Collaborative Positioning

Multi-agents Collaborative Positioning



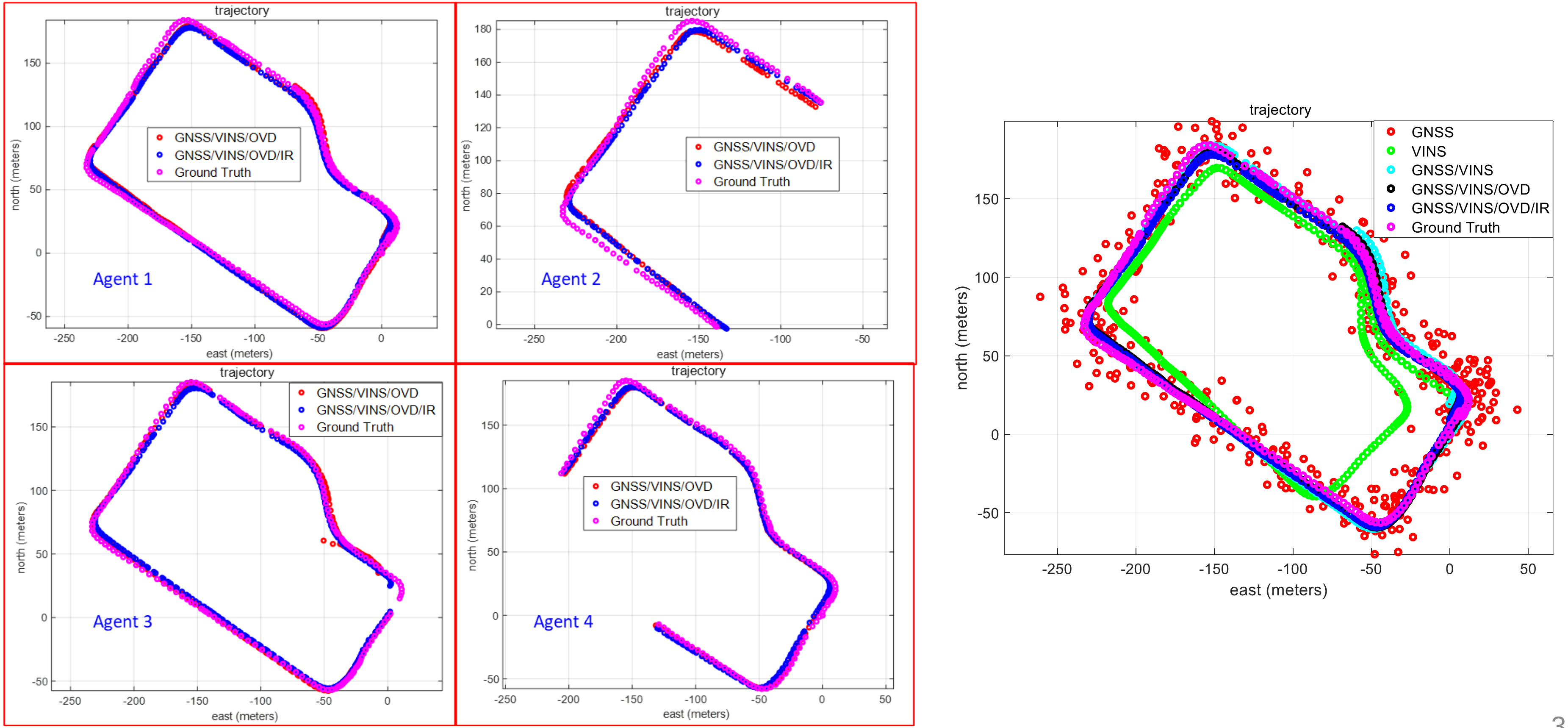
Multi-agents Collaborative Positioning



Multi-agents Collaborative Positioning

Methods		Agent 1		Agent 2		Agent 3		Agent 4	
		MEAN	STD	MEAN	STD	MEAN	STD	MEAN	STD
Single Agent Positioning	GNSS	12.49 m	7.09 m	11.5 m	6.80 m	13.1 m	6.77	12.07 m	6.37 m
	VINS (RPE)	0.625 m	0.52 m	0.49 m	0.36 m	0.45 m	0.64 m	0.87 m	0.92 m
	VINS (APE)	16.81 m	6.76 m	7.99 m	2.40 m	23.2 m	12.6 m	21.59 m	8.62 m
	GNSS/VINS	5.49 m	3.33 m	6.13 m	3.92 m	5.01 m	3.45 m	5.71 m	3.85 m
Multi-Agents Positioning	GNSS/VINS/OVD	4.22 m	1.95 m	5.33 m	1.70 m	4.39 m	4.86 m	3.94 m	2.01 m
	GNSS/VINS/OVD /IR	3.81 m	1.82 m	3.66 m	1.16 m	3.42 m	1.58 m	3.63 m	1.82 m

Multi-agents Collaborative Positioning



Coordinate Systems

Coordinate Systems

- A significant problem to overcome when using a navigation system is the fact that there are a great number of different coordinate systems worldwide.
- As a result, the position measured and calculated does not always correspond with one's supposed position.

Big Ben, London, UK

Lat./Long./Alt.	51.5007° N	0.1246° W	72m
X/Y/Z-WGS84	3978622.9m	8652.2m	4968467.2m
E/N/U relative to Westminster Station	45.3m	-32.5m	68.9m
front/right/up (relative to your body)	-30.8m	20.3m	69.9m

Why we need the navigation coordinate system?

Scenario: A planned flight from Hong Kong airport to the Los Angeles.
Question: How can I know the distance from current (e.g., waypoint j) to the destination?

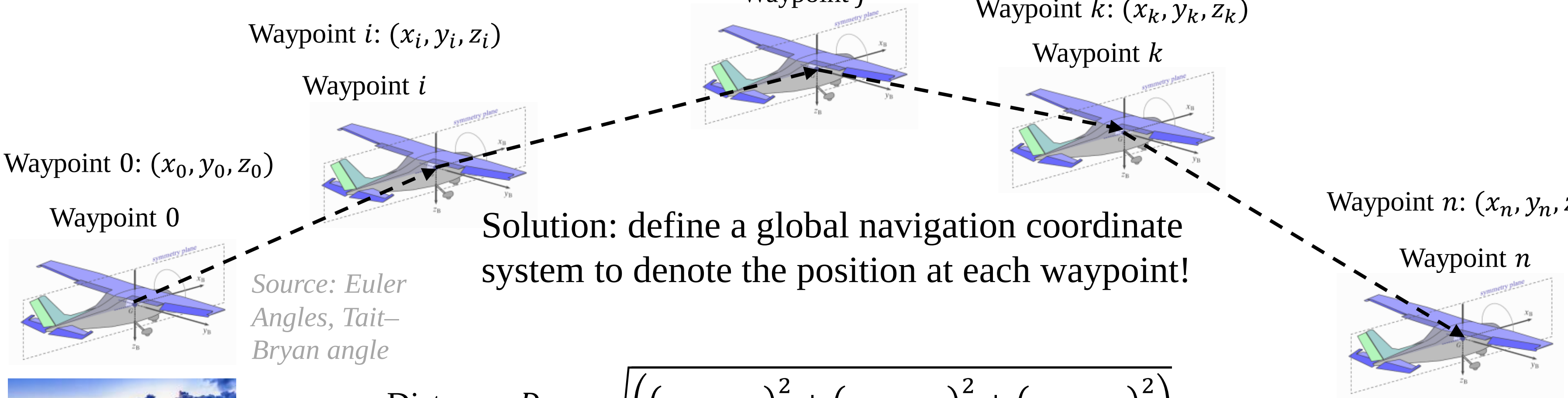
Waypoint j: (x_j, y_j, z_j)

Waypoint i: (x_i, y_i, z_i)

Waypoint k: (x_k, y_k, z_k)

Waypoint 0: (x_0, y_0, z_0)

Waypoint n: (x_n, y_n, z_n)



Source: Euler Angles, Tait–Bryan angle

Solution: define a global navigation coordinate system to denote the position at each waypoint!

Hong Kong

$$\text{Distance: } D_{jn} = \sqrt{\left((x_j - x_n)^2 + (y_j - y_n)^2 + (z_j - z_n)^2\right)}$$

Question: how to define the global coordinate system applicable for both HK and Los Angeles?

Coordinate Systems

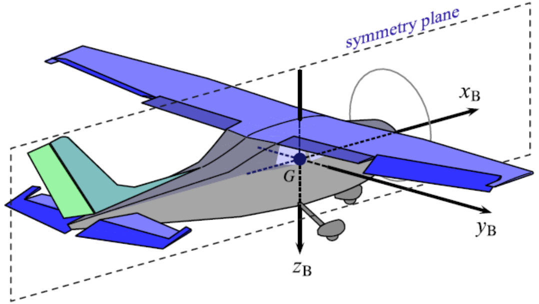
Any navigation problem thus involves at least two coordinate frames. One describing the body frame and one as the reference frame. Any two coordinate frames may have any relative orientation, known as **attitude**.

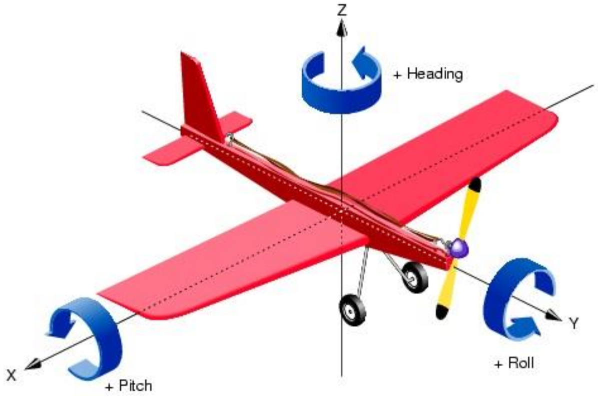
The main coordinate systems used in navigation: *Earth-centered inertial (ECI), Earth-centered Earth-fixed (ECEF) and body frames*

In physics, any coordinate frame that does not accelerate or rotate with respect to the rest of the Universe is an *inertial frame*.

Body Navigation Frame - Body

- It represents the orientation of the body to which it is connected.
- Its origin is described by the aerial vehicle (sometime is the center gravity of sensors)
- The axes are
- X^b –axis: pointing towards the right to the direction of motion
- Y^b –axis: pointing towards the front (in the direction of motion)
- Z^b–axis: pointing up to complete the orthogonal right-hand
  Applications
  - Attitude derivation
  - Flight control system
  - Simultaneous localization and mapping (SLAM)

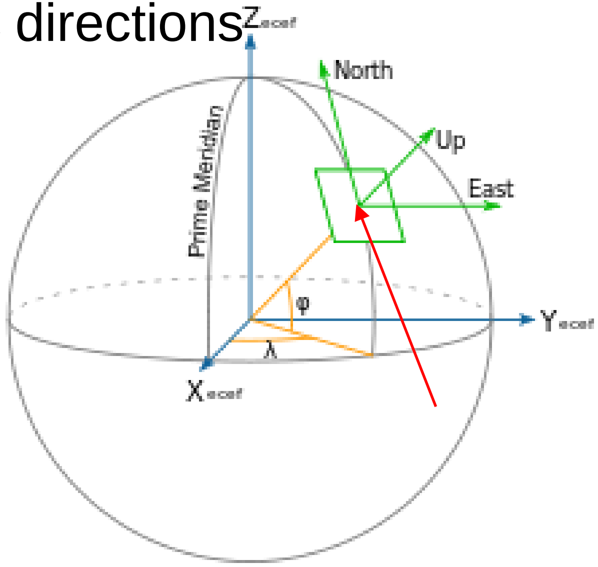




Local Navigation Frame - ENU

Origin defined by User

- Local (Relative) Frame.
- Its origin is described by a navigation object (usually in ECEF).
- The axes are aligned with topographic directions North, East and Vertical (Up).

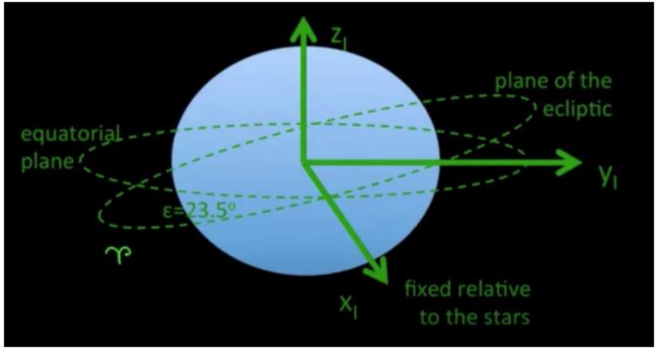


- (E, N, U) is used to denote position
- Applications
  - Robotic Navigation
    Since the user wants to know his/her position/attitude relative to the north, east and vertical direction.

https://en.wikipedia.org/wiki/Axes_conventions

Earth Centered Inertial (ECI)

The origin is at the center of the mass of the Earth and whose axes are pointing in **fixed directions** with respect to the stars, which does not rotates with the earth.

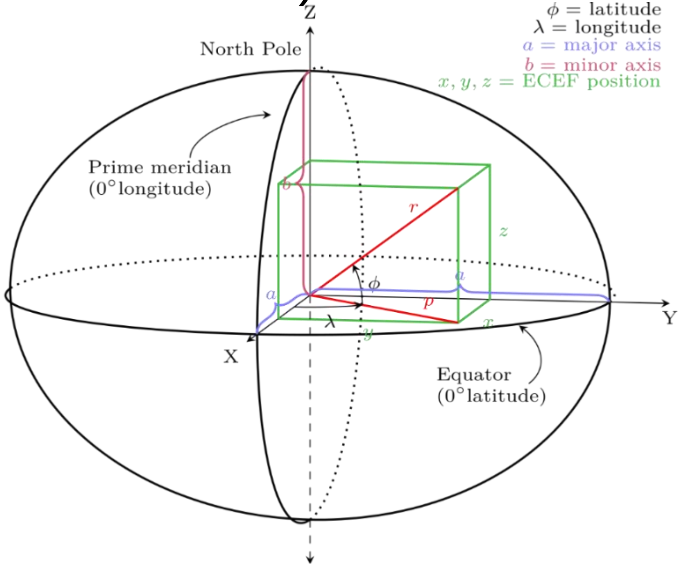


- The *z*-axis always points along the Earth's axis of rotation from the frame's origin at the center of mass to the true north pole (not the magnetic pole).
- The *x*- and *y*-axes lie within the equatorial plane, but do not rotate with the Earth. +*x*-axis is permanently fixed in a particular direction relative to the celestial sphere.
- The *y*-axis points 90° ahead of the *x*-axis in the direction of the Earth's rotation.

Earth-Centered, Earth-Fixed - ECEF

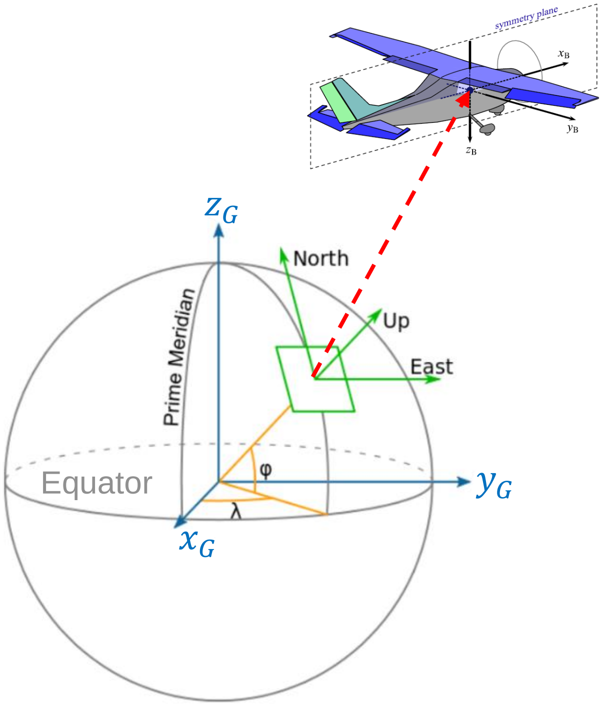
IERS*: Earth Rotation and Reference Systems Service

- Similar to ECI except all axes remain fixed w.r.t Earth.
- Z-axis is rotated with the Earth spin axis (same with ECI).
- X-axis points to 0º longitude defined by IERS.
- Y-axis is orthogonal with X-axis.
- (0,0,0) means Center of Earth Mass
- Both (X, Y, Z) and (Lat, Lon, Alt) can be used to denote positions
- Applications
  - GPS positioning



https://en.wikipedia.org/wiki/ECEF

Body && ENU && ECEF



λ: Longitude
φ: Latitude

Earth-centered earth-fixed (ECEF) coordinate system, $\mathbf{C}_G(x_G, y_G, z_G)$

- Origin at the center of mass of earth
- x_G extends through the intersection of the prime meridian Greenwich and the equator.
- z_G points towards geographical north
- y_G completes the right handset of coordinates axis

East-North-Up (ENU) coordinate system, $C_L(x_L, y_L, z_L)$

- Local tangent plane coordinates based on **a selected reference point**
- The east axis is labeled x, the north y and the up z.
- Rotate with the earth
- Useful to describe motion of objects on earth surface.

Representation of Position and Attitude

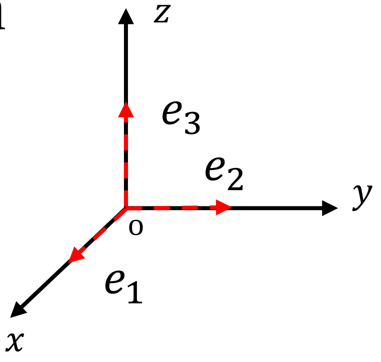
Prerequisite and Notation

Prerequisite :

- Matrix and vector calculation
- Unit orthogonal basis of coordinate system

Notation :

- Matrix with bold upper case (**R**)
- Vector with bold lower case (**v**)
- Scalar with low case italic (*k*)



Unit orthogonal basis $[\mathbf{e}_1 \quad \mathbf{e}_2 \quad \mathbf{e}_3]$, which satisfying

$$[\mathbf{e}_1 \quad \mathbf{e}_2 \quad \mathbf{e}_3]\begin{bmatrix}\mathbf{e}_1\\ \mathbf{e}_2\\ \mathbf{e}_3\end{bmatrix} = [1]$$

Mathematical Foundations- Matrix Calculations

$$\begin{cases} 3x + 4y + z = 9 \\ 2x - 6y + 3z = 8 \\ 7x + 5y - 7z = 1 \end{cases}$$

3 linear equations with 3 unknowns

Stack into matrix form →

$$\begin{bmatrix} 3 & 4 & 1 \\ 2 & -6 & 3 \\ 7 & 5 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ 1 \end{bmatrix} \rightarrow \mathbf{A}\boldsymbol{x} = \boldsymbol{b}$$

Matrix **A** Vector $\boldsymbol{x}$ Vector $\boldsymbol{b}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -6 & 3 \\ 7 & 5 & -7 \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ 8 \\ 1 \end{bmatrix} \rightarrow \boldsymbol{x} = \mathbf{A}^{-1}\boldsymbol{b}$$

Condition for matrix multiplication: $\mathbf{AB}$ → The columns of $\mathbf{A}$ should be the same as the row of $\mathbf{B}$

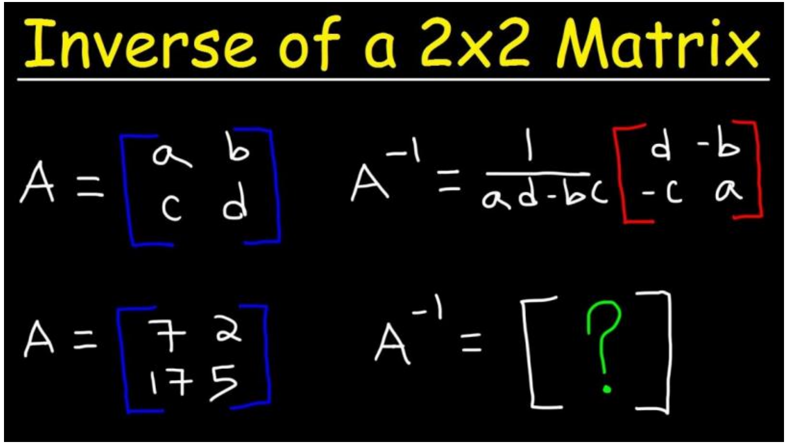
Condition for matrix inverse: $\mathbf{A}^{-1}$ → The $\mathbf{A}$ is an full rank matrix

Transpose of matrix: $\mathbf{A}^{\mathrm{T}}$ Identity matrix: $\mathbf{I}_{n\times n}$

Mathematical Foundations- Matrix Calculations

Add operation between matrix: $\mathbf{A}+\mathbf{B}$

$$\begin{bmatrix}3 & 4 & 1\\ 2 & -6 & 3\\ 7 & 5 & -7\end{bmatrix}+\begin{bmatrix}0 & 4 & 1\\ 2 & 1 & 1\\ 7 & 5 & 2\end{bmatrix}=\begin{bmatrix}3 & 8 & 2\\ 4 & -5 & 4\\ 14 & 10 & -5\end{bmatrix}$$



Multiply operation between matrix: $\mathbf{A}*\mathbf{B}$

$$\begin{bmatrix}3 & 4 & 1\\ 2 & -6 & 3\\ 7 & 5 & -7\end{bmatrix}\begin{bmatrix}0 & 4 & 1\\ 2 & 1 & 1\\ 7 & 5 & 2\end{bmatrix}=\begin{bmatrix}15 & 21 & 9\\ 9 & 17 & 2\\ -39 & -2 & -2\end{bmatrix}$$

Take a try on this ☺

Transpose of matrix: $\mathbf{A}^{\mathrm{T}}$

$$\begin{bmatrix}3 & 4 & 1\\ 2 & -6 & 3\\ 7 & 5 & -7\end{bmatrix}^{\mathrm{T}}=\begin{bmatrix}3 & 2 & 7\\ 4 & -6 & 5\\ 1 & 3 & -7\end{bmatrix}$$

Try Yourself

Add operation between matrix: $\mathbf{A}+\mathbf{B}$

$$\begin{bmatrix}1 & 3 & 2\\ 1 & -1 & 0\\ 2 & 6 & -2\end{bmatrix}+\begin{bmatrix}4 & 2 & 3\\ 2 & 1 & 5\\ 2 & 3 & 7\end{bmatrix}=?$$

Results:

$$\begin{bmatrix}5 & 5 & 5\\ 3 & 0 & 5\\ 4 & 9 & 5\end{bmatrix}$$

Multiply operation between matrix: $\mathbf{A}*\mathbf{B}$

$$\begin{bmatrix}1 & 2 & 3\\ 3 & -5 & 2\\ 6 & 2 & -1\end{bmatrix}\begin{bmatrix}0 & 2 & 1\\ 1 & 3 & -1\\ 5 & 6 & 2\end{bmatrix}=?$$

$$\begin{bmatrix}17 & 26 & 5\\ 5 & 3 & 12\\ -3 & 12 & 2\end{bmatrix}$$

Transpose of matrix: $\mathbf{A}^{\mathrm{T}}$

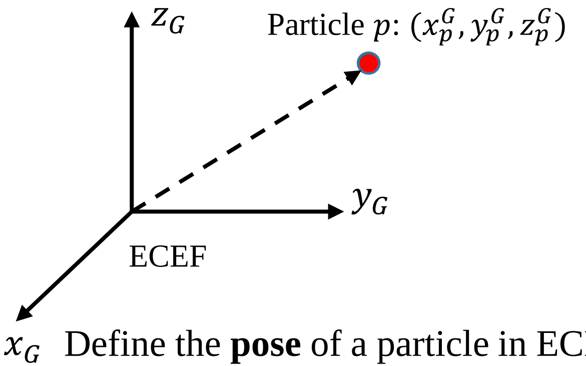
$$\begin{bmatrix}3 & 2 & 5\\ 2 & -1 & 4\\ 5 & 7 & -1\end{bmatrix}^{\mathrm{T}}=?$$

$$\begin{bmatrix}3 & 2 & 5\\ 2 & -1 & 7\\ 5 & 4 & -1\end{bmatrix}$$

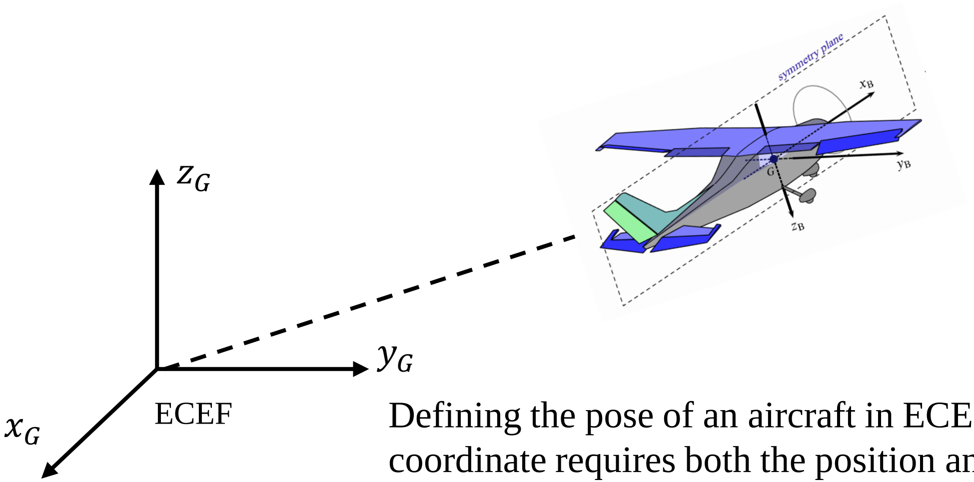
Pose Representation of UAV in Space

Scenario: A planned flight from Hong Kong airport to the Los Angeles.

Question: How to define the pose of a flight in the space?



Define the **pose** of a particle in ECEF coordinate as $\left(x_p^G, y_p^G, z_p^G\right)$. The G denotes the ECEF global coordinate system.



Defining the pose of an aircraft in ECEF coordinate requires both the position and orientation as $(x_p^G, y_p^G, z_p^G, \phi_p^G, \theta_p^G, \psi_p^G)$

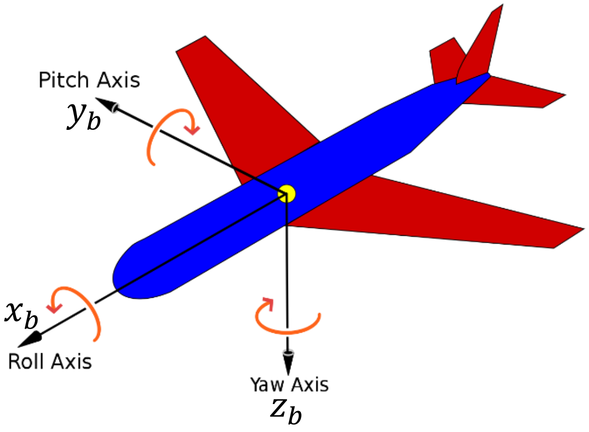
position (x_p^G, y_p^G, z_p^G) + **orientation** ($\phi_p^G, \theta_p^G, \psi_p^G$) → **Pose**

Pose Representation of UAV in Space

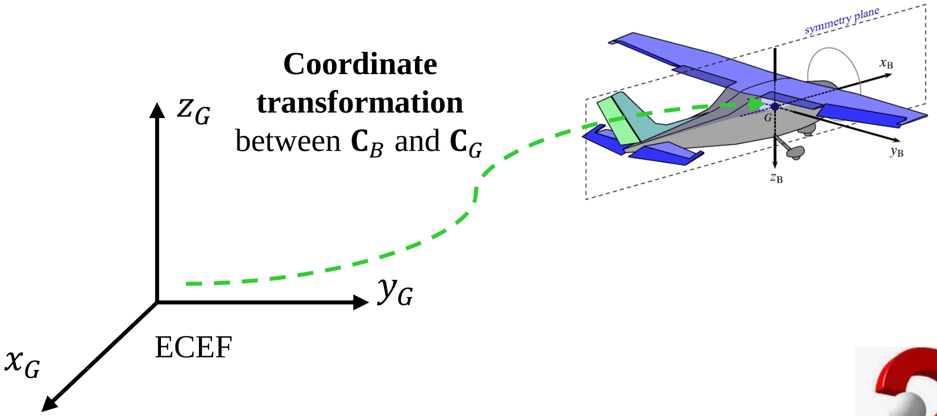
Pose of an aircraft in ECEF coordinate $(x_p^G, y_p^G, z_p^G, \phi_p^G, \theta_p^G, \psi_p^G)$:

- x_p^G, y_p^G, z_p^G, **position** in ECEF frame
- $\phi_p^G, \theta_p^G, \psi_p^G$, the **orientation** (*roll*, *pitch* and *yaw angle*).

Since the body-fixed coordinate $\mathbf{C}_B$ is fixed on the flight mechanics. The **coordinate transformation** between $\mathbf{C}_B$ and $\mathbf{C}_G$ represents the **pose of the flight mechanics in the ECEF frame**!



Source: Euler Angles, Tait–Bryan angle



How to denote the coordinate transformation?

Rotation Representation with Matrix: Derivation from Orthogonal Basis

Define the unit orthogonal basis of $\mathbf{C}_G$ as $[\mathbf{e}_x^G, \mathbf{e}_y^G, \mathbf{e}_z^G]$ and the coordinate of vector $\mathbf{a}$ as $[a_x^G, a_y^G, a_z^G]$.

Define the unit orthogonal basis of $\mathbf{C}_B$ as $[\mathbf{e}_x^B, \mathbf{e}_y^B, \mathbf{e}_z^B]$ and the coordinate of vector $\mathbf{a}$ as $[a_x^B, a_y^B, a_z^B]$.

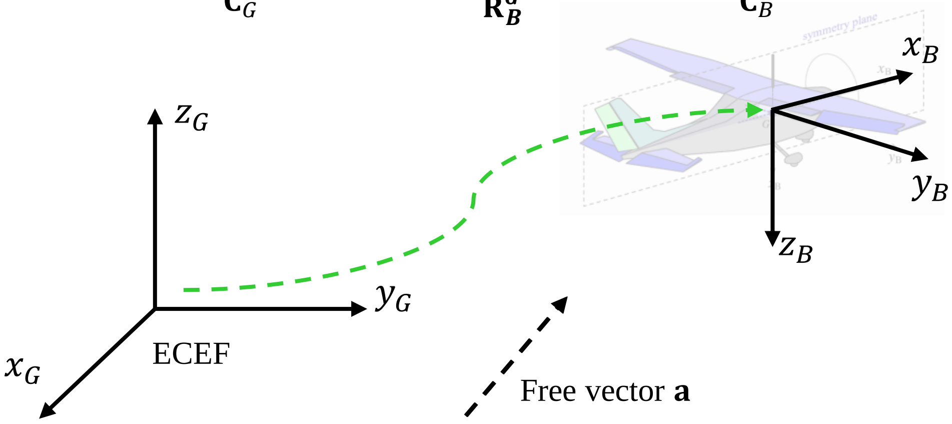
Since the vector $\mathbf{a}$ itself is **constant despite of the representation** in different coordinate systems. We have

$$[\mathbf{e}_x^G, \mathbf{e}_y^G, \mathbf{e}_z^G]\begin{bmatrix} a_x^G \\ a_y^G \\ a_z^G \end{bmatrix} = [\mathbf{e}_x^B, \mathbf{e}_y^B, \mathbf{e}_z^B]\begin{bmatrix} a_x^B \\ a_y^B \\ a_z^B \end{bmatrix}$$

Multiply the $[\mathbf{e}_x^G, \mathbf{e}_y^G, \mathbf{e}_z^G]^T$ to both sides, we get

$$\begin{bmatrix} a_x^G \\ a_y^G \\ a_z^G \end{bmatrix} = \begin{bmatrix} \mathbf{e}_x^G\mathbf{e}_x^B & \mathbf{e}_x^G\mathbf{e}_y^B & \mathbf{e}_x^G\mathbf{e}_z^B \\ \mathbf{e}_y^G\mathbf{e}_x^B & \mathbf{e}_y^G\mathbf{e}_y^B & \mathbf{e}_y^G\mathbf{e}_z^B \\ \mathbf{e}_z^G\mathbf{e}_x^B & \mathbf{e}_z^G\mathbf{e}_y^B & \mathbf{e}_z^G\mathbf{e}_z^B \end{bmatrix}\begin{bmatrix} a_x^B \\ a_y^B \\ a_z^B \end{bmatrix}$$

Position in $\mathbf{C}_G$ — Rotation Matrix $\mathbf{R}_B^G$ — Position in $\mathbf{C}_B$



The rotation between two coordinate systems can be represented by the rotation matrix $\mathbf{R}_B^G$!

Rotation and Position Representation

Given

- The position of a particle **a** in the body-fixed coordinate as (a_x^B, a_y^B, a_z^B)
- The transformation between between $\mathbf{C}_B$ and $\mathbf{C}_G$ as rotation matrix $\mathbf{R}_B^G$ and translation vector $\mathbf{t}_B^G(x_B^G, y_B^G, z_B^G)$

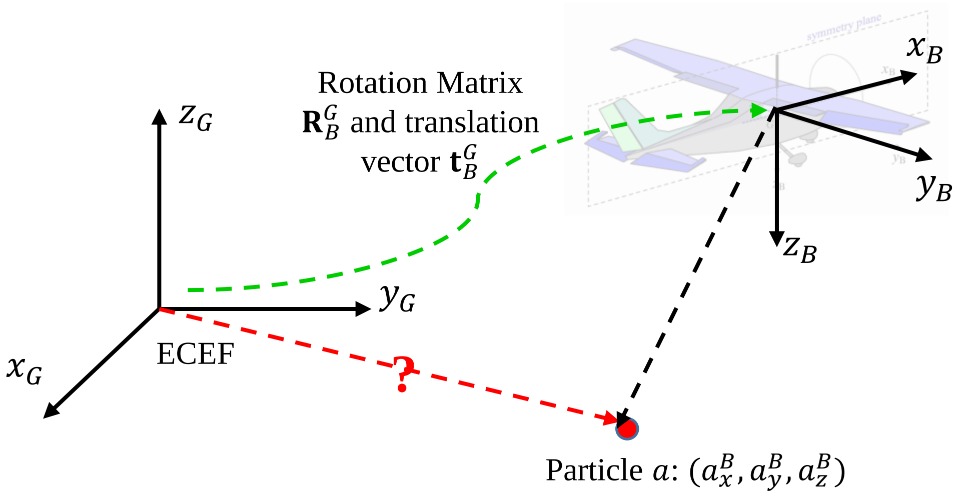
Question:

- Calculate the coordinate of particle **a** in the coordinate $\mathbf{C}_G$.

Considering both the rotation and position, we get

$$\begin{bmatrix} a_x^G \\ a_y^G \\ a_z^G \end{bmatrix} = \begin{bmatrix} e_x^G e_x^B & e_x^G e_y^B & e_x^G e_z^B \\ e_y^G e_x^B & e_y^G e_y^B & e_y^G e_z^B \\ e_z^G e_x^B & e_z^G e_y^B & e_z^G e_z^B \end{bmatrix} \begin{bmatrix} a_x^B \\ a_y^B \\ a_z^B \end{bmatrix} + \begin{bmatrix} x_B^G \\ y_B^G \\ z_B^G \end{bmatrix}$$

Position in $\mathbf{C}_G$ | **Rotation Matrix $\mathbf{R}_B^G$** | **Position in $\mathbf{C}_B$** | **$\mathbf{t}_B^G$,translation between $\mathbf{C}_G$ and $\mathbf{C}_B$**



The $\mathbf{R}_B^G$ represent the orientation and the $\mathbf{t}_B^G$ represents the position of the flight mechanic in the ECEF coordinate system!

Question

Given

- The position of a particle **a** in the ECEF coordinate as (a_x^G, a_y^G, a_z^G)
- The transformation between between $\mathbf{C}_B$ and $\mathbf{C}_G$ as rotation matrix $\mathbf{R}_B^G$ and translation vector $\mathbf{t}_B^G$

Question:

- Calculate the coordinate of particle **a** in the coordinate $\mathbf{C}_B$.

Solution:

Since we have $\begin{bmatrix} a_x^G \\ a_y^G \\ a_z^G \end{bmatrix} = \mathbf{R}_B^G \begin{bmatrix} a_x^B \\ a_y^B \\ a_z^B \end{bmatrix} + \mathbf{t}_B^G$,

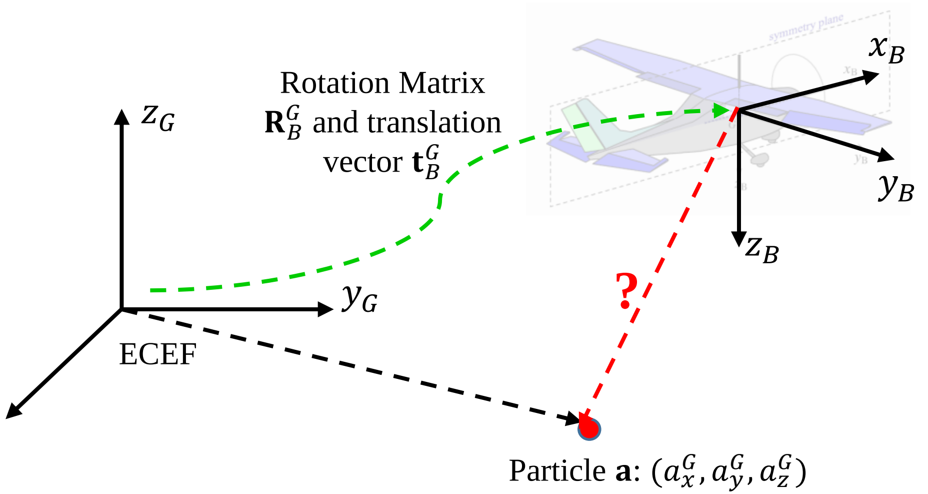
Therefore, we have

$$\begin{bmatrix} a_x^G \\ a_y^G \\ a_z^G \end{bmatrix} - \mathbf{t}_B^G = \mathbf{R}_B^G \begin{bmatrix} a_x^B \\ a_y^B \\ a_z^B \end{bmatrix},$$

Solution:

Multiple ${\mathbf{R}_B^G}^{-1}$ on both sides, we get

$$\begin{bmatrix} a_x^B \\ a_y^B \\ a_z^B \end{bmatrix} = {\mathbf{R}_B^G}^{-1} \left(\begin{bmatrix} a_x^G \\ a_y^G \\ a_z^G \end{bmatrix} - \mathbf{t}_B^G \right),$$



Q&A

Thank you for your attention ☺

Q&A

Dr. Weisong Wen

If you have any questions or inquiries, please feel free to contact me.

Email: welson.wen@polyu.edu.hk